EE2013

NON-LINEAR CIRCUIT ANALYSIS

LECTURE 18: ACTIVE OPAMP FILTER DESIGN

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Coordinator: Prof. Pádraig Cantillon-Murphy

LECTURE SCHEDULE

Thursdays 11am-1pm
(with short break)

Monday 9am-10am slot not used!

LECTURE NOTES

https://www.jaeger.ie/ee2013/lec17

Uploaded before lecture takes place

QUESTIONS?

Just ask whenever it comes to you!

OR:

anthony.wall@mcci.ie on Email, Teams or Canvas

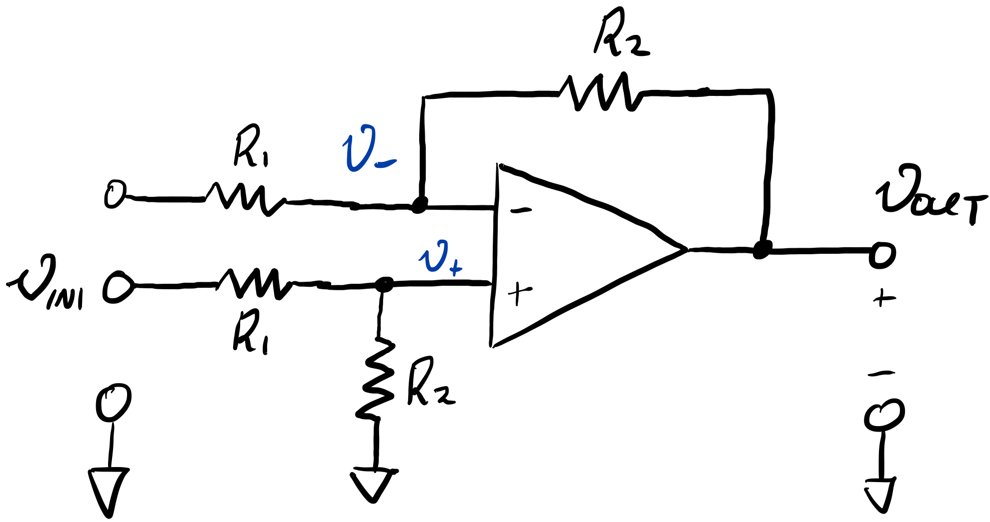
1 Review from Last Time

1.1 The Differential Amplifier

$$v_- \simeq v_+$$

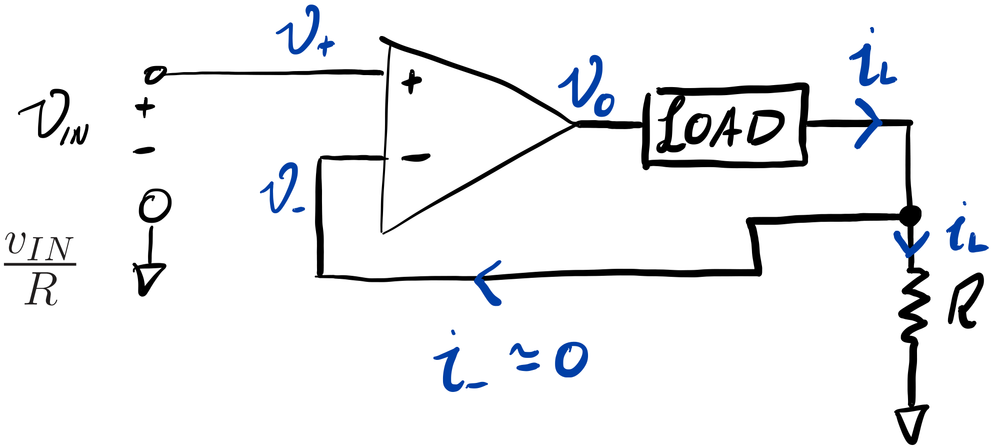
$$\hookrightarrow v_- \simeq v_{IN1} \left(\frac{R_2}{R_1 + R_2}\right)$$

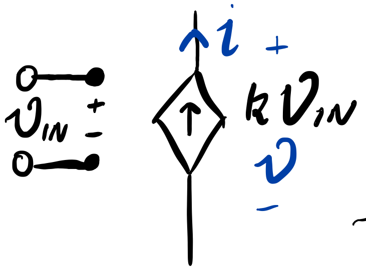
$$v_O = \frac{R_2}{R_1}(v_1 - v_2)$$

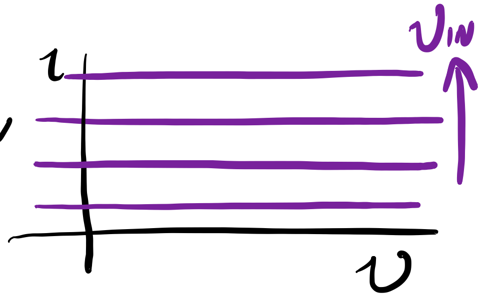


1.2 The Op Amp Current Source

$$i \approx \frac{v_{IN}}{R}$$

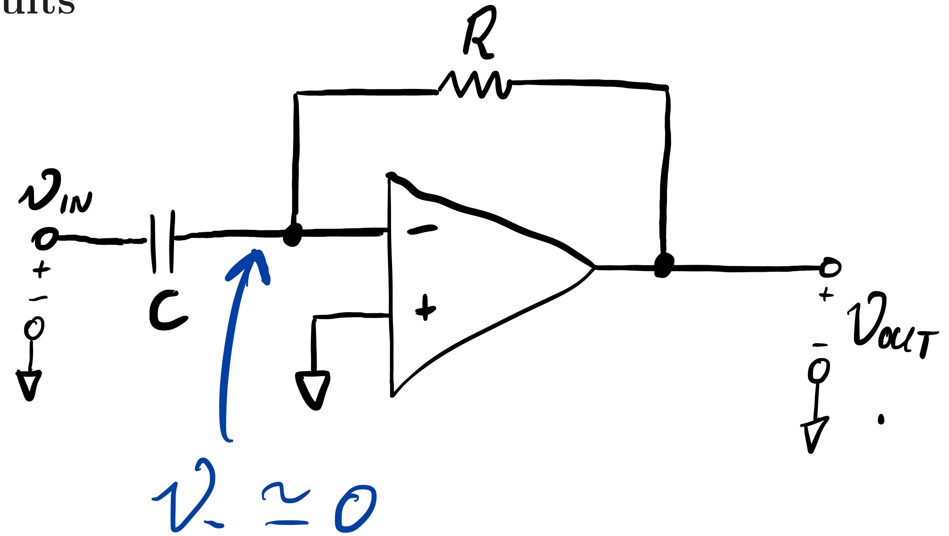




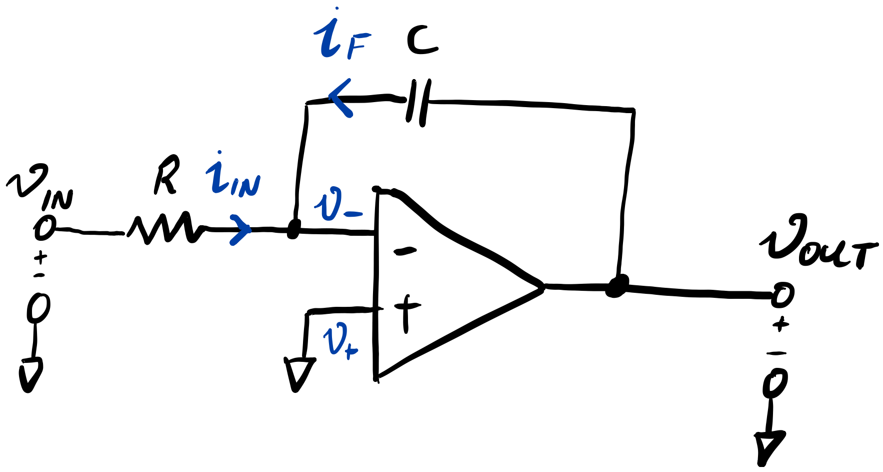


1.3 Op Amp Capacitor Circuits

$$v_O = -RC\frac{dv_{IN}}{dt}$$

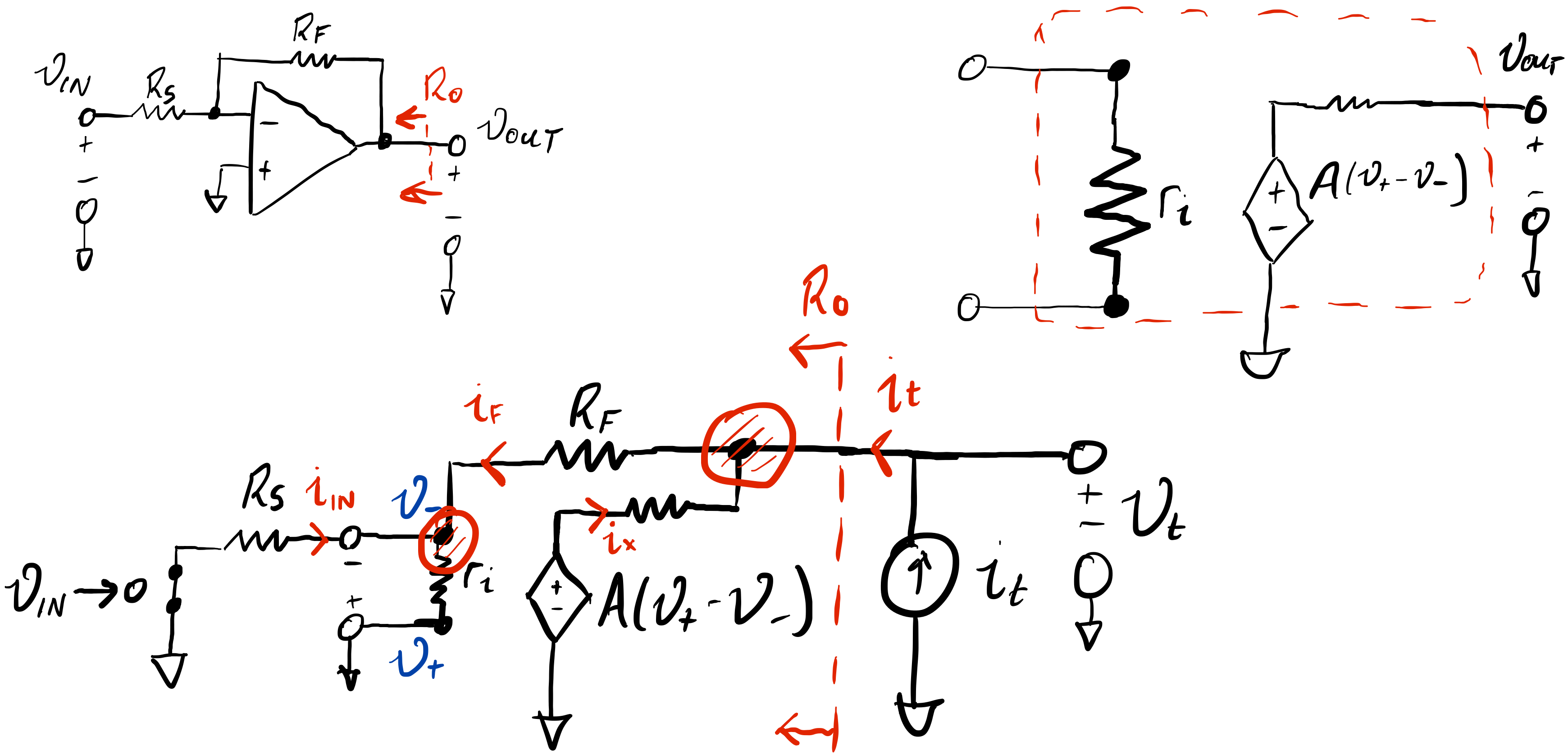


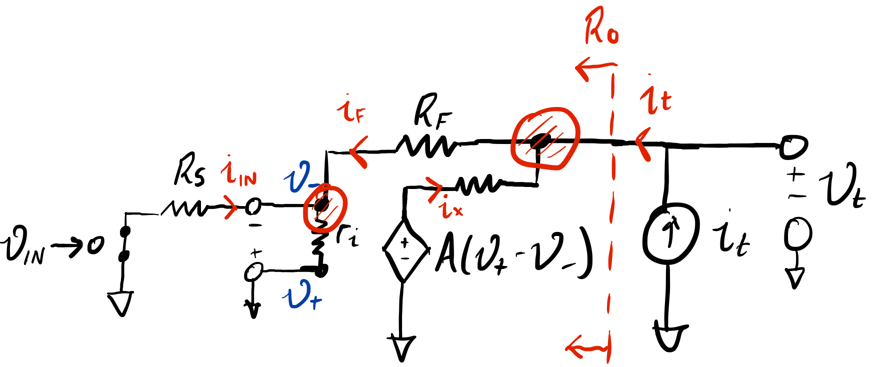
$$v_O = -\frac{1}{RC}\int_0^t v_{IN}dt + v_C(0)$$



2 Inverting Op Amp Resistance

Consider the inverting op amp circuit below. Ideally the output resistance (*i.e.*, the Thevenin equivalent resistance seen at the output terminals of the circuit) should be small to enable the circuit to drive subsequent stages. To calculate its value, we need to improve our op amp model by adding a (large) input resistance, r_i, and a (small) output resistance, r_o.





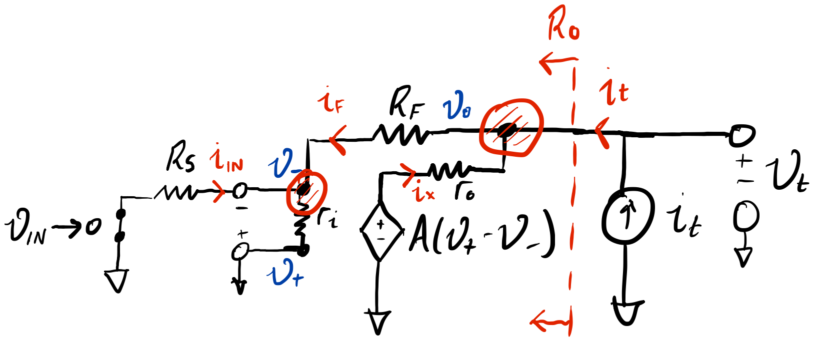
2.1 Output Equivalent Resistance, R_0

Because of the dependent source in the model, we need to apply a test source at the output terminal to calculate the output resistance of the circuit where $R_O = \frac{v_o}{i_t}$.

For simplicity we will employ conductances instead of resistances:

$$\frac{1}{r_i} = g_i \quad , \quad \frac{1}{r_o} = g_o \quad , \quad \frac{1}{R_F} = G_F \quad , \quad \frac{1}{R_S} = G_S$$

Clearly, $v^+ = 0 \Rightarrow v_- \simeq 0$



Applying KCL at node v^-: $\sum i_{IN} - \sum i_{OUT} = 0$

$$(0 - v_-)G_S + (v_o - v_-)G_F - (v_-)g_i = 0$$

$$v^- = \frac{G_F v_o}{G_S + G_F + \cancel{g_i}\,0} = \frac{G_F v_O}{G_S + G_F}$$ ....for $g_i \to 0$

Applying KCL at node v_o: $\sum i_{IN} - \sum i_{OUT} = 0$

$$i_t + [A(v_+ - v_-) - v_o]g_o - (v_o - v_-)G_F = 0$$

Substituting for v^+ and v^-:

$$-Ag_o\frac{G_F v_o}{G_S + G_F} - v_o g_o + i_t - v_o G_F + \frac{G_F^2 v_o}{G_S + G_F} = 0$$

Simplifying:

$$-Ag_o\frac{G_F v_o}{G_S + G_F} - v_o g_o + i_t + v_o\left(\frac{-G_S G_F - G_F^2 + G_F^2}{G_S + G_F}\right) = 0$$

$$G_o = \frac{i_t}{v_o} = \underbrace{\frac{A g_o G_F}{G_S + G_F}}_{\text{OpAmp Feedback}} + \underbrace{g_o}_{\text{Output Conductance}} + \underbrace{\cancel{\frac{G_S G_F}{G_S + G_F}}}_{\text{Feedback Resistor Combination}} \to 0$$

For large A:

$$G_O \approx g_o\left(\frac{AG_F}{G_S + G_F} + 1\right) = g_o\left(1 + \underbrace{\frac{AR_S}{R_S + R_F}}_{\text{Internal Loop Gain}}\right)$$

Output Conductance

Internal Input Conductance

Output Conductance

$$G_O \approx g_o(\frac{AG_F}{G_S + G_F} + 1) = g_o(1 + \underbrace{\frac{AR_S}{R_S + R_F}}_{\text{Internal Loop Gain}})$$

Internal Input Conductance

This, in fact, is a general result for any linear circuit in which the feedback resistor is sampling the output node voltage (rather than the output current).
Where A dominates the loop gain such at $A \gg 1 + \frac{R_F}{R_S}$, then:

$$R_o \approx \frac{r_o}{A\frac{R_S}{R_S+R_F}} \approx \frac{r_o(R_S + R_F)}{AR_S}$$

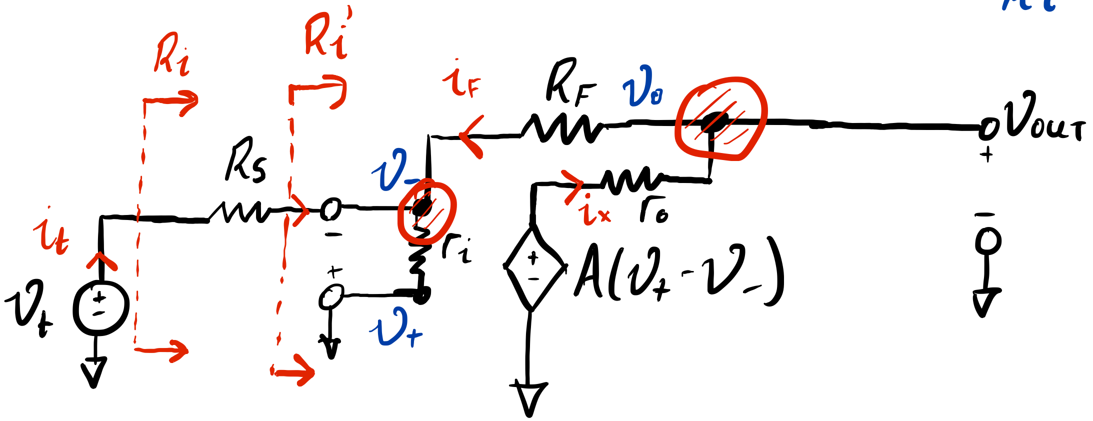
Typically, $r_o \approx 1000\ \Omega$ and $R_o \lesssim 0.1\ \Omega$.

2.2 Input Equivalent Resistance, R_i

To calculate the input equivalent resistance, we add a test voltage source at the input. The analysis is simplified if we ignore R_S in the analysis ($R_S \to 0$) but add it to the final result.

$$R_i = \frac{v_t}{i_t}$$

$$R_i' = R_i + R_S$$



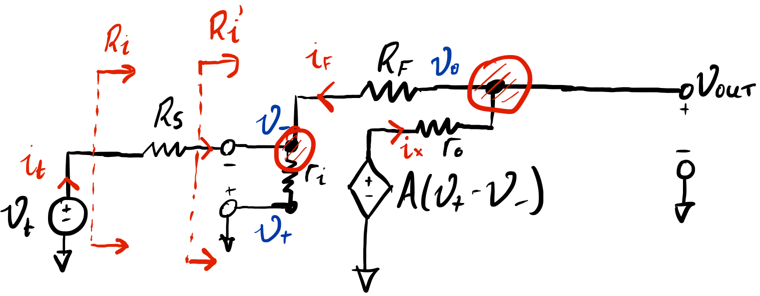
Clearly: $v^+ = 0$

$v^- = v_t$ for $R_S \to 0$

Applying KCL at node v^-: $\sum i_{IN} - \sum i_{OUT} = 0$

$$i_t + \frac{v_o - v_-}{R_F} - \frac{v_-}{r_i} = 0$$

For R_i', Let $v_- = v_t$



KCL @ v_{OUT}: $i_x - i_F = 0$

$$\left(\frac{A(\overset{0}{\cancel{v_+}} - v_-) - v_o}{r_o}\right) - \left(\frac{v_o - v_-}{R_F}\right) = 0$$

Substituting for $\underset{0 =}{v^+}$ and v^-: $= v_t$

$$i_t - \frac{v_t}{r_i} - \frac{v_t}{R_F + r_o} - \frac{Av_t}{R_F + r_o} = 0$$

$$\Rightarrow \frac{i_t}{v_t} = G_i' = \frac{1}{R_i'} = \frac{1}{r_i} + \frac{1}{R_F + r_o} + \underbrace{\frac{A}{R_F + r_o}}_{\text{Dominant Term}}$$

Internal OpAmp Conductance

For large A:

$$R_i' = \frac{1}{G_i'} \approx \frac{R_F + ro}{A}$$

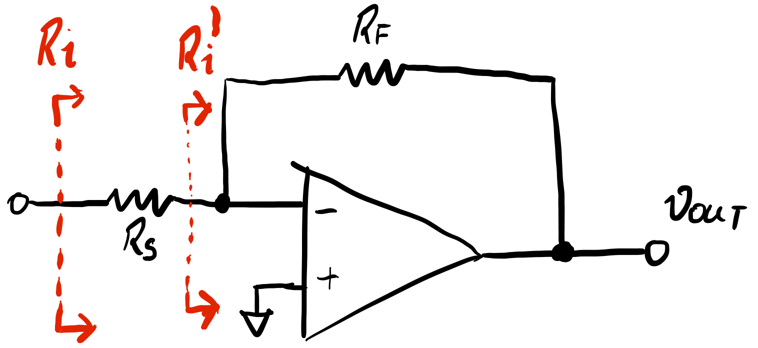
For large $A \sim 10^5$, the input terminal resistance will be very small ($R_i \lesssim 1\Omega$). This is intuitive because even for small input voltages ($v_{IN} \lesssim 1$ mV), the output voltage, v_O, is $-A$ times larger. So this large voltage has to be dropped entirely across the feedback resistor, R_F, which carries relatively large currents even when the input voltage is small. This large feedback current for small input voltage indicates that the effective input resistance is very small.

The complete input resistance of the circuit includes the effect of R_S:

$$R_i = R_S + R_i' = R_S + \frac{R_F + r_o}{A}$$

Since R_i' is small, it is R_S which usually dominates the input resistance:

$R_i \simeq R_S$

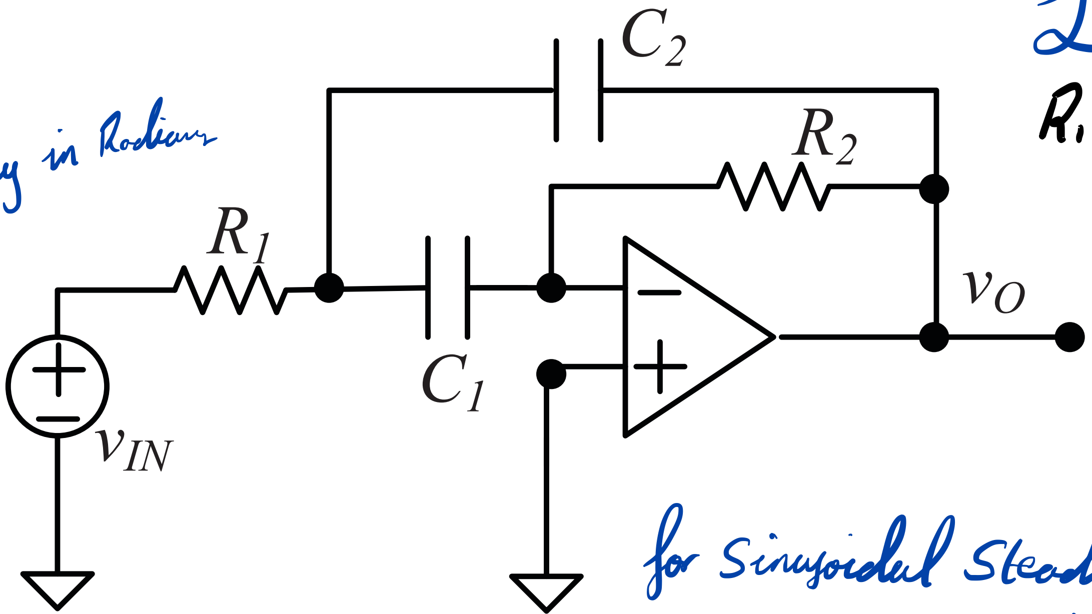


3 Active Op Amp Filters

Inductors are not ideal elements for integrated circuits because they are difficult to fabricate on silicon and can introduce system variability. Therefore, very large scale integrated (VLSI) circuit designers will seek to eliminate inductors with *active* elements such as transistors and op amps for low power applications. The following example illustrates this in the case of an *active* bandpass filter which we previously saw implemented as a passive RLC combination in EE2012.

$j = \sqrt{-1}$

ω = frequency in Radians



Laplace Representation:

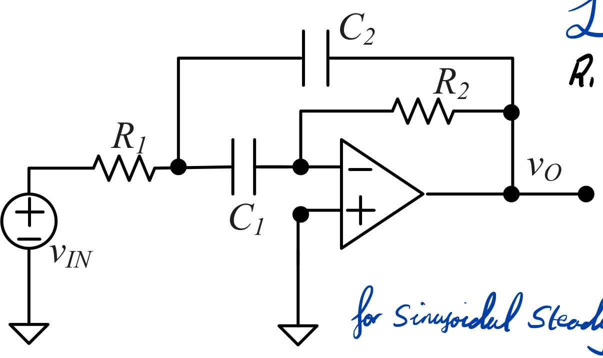
$R_1 \rightarrow R_1$

$R_2 \rightarrow R_2$

$C_1 \rightarrow \frac{1}{sC_1}$

$C_2 \rightarrow \frac{1}{sC_2}$

for Sinusoidal Steady State: $s = j\omega$



Laplace Representation:

$R_1 \rightarrow R_1$

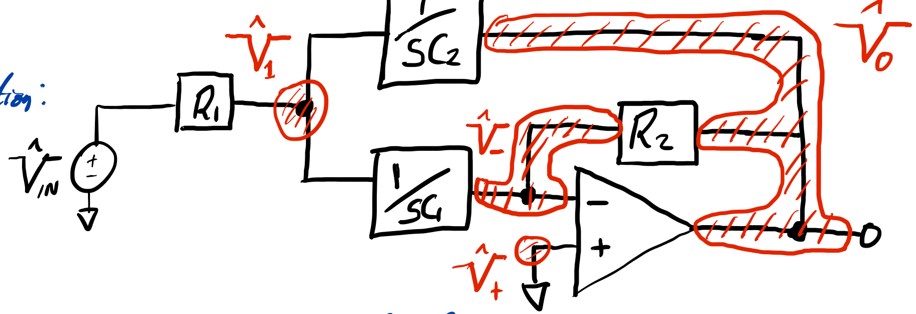
$R_2 \rightarrow R_2$

$C_1 \rightarrow \frac{1}{sC_1}$

$C_2 \rightarrow \frac{1}{sC_2}$

for Sinusoidal Steady State: $s = j\omega$

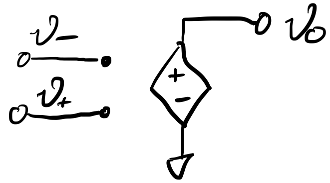
To analyse the circuit, nodal analysis is used using the complex coefficients :



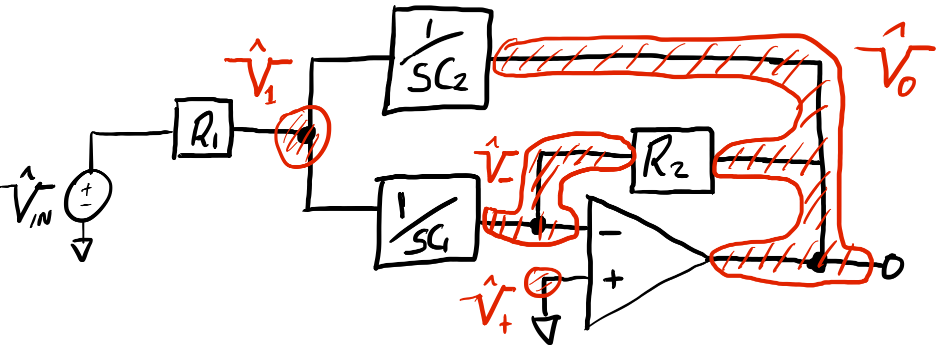
Assume Ideal opamps:

$\hat{V}_+ = 0$

$\hookrightarrow \hat{V}_- \simeq \hat{V}_+ = 0$



$G_1 = 1/R_1 \quad G_2 = 1/R_2$



Applying KCL at node $\hat{V}_1$: $\sum i_{IN} - \sum i_{OUT} = 0$

$$(\hat{V}_{IN} - \hat{V}_1)G_1 + (\hat{V}_o - \hat{V}_1)sC_2 - \hat{V}_1 sC_1 = 0$$

where $G_1 = 1/R_1$ and $G_2 = 1/R_2$

Applying KCL at node $\hat{V}^-$ (where $\hat{V}^- = 0$):

$$\sum i_{IN} - \sum i_{OUT} = \hat{V}_1 sC_1 + \hat{V}_o G_2 = 0$$

$$\hat{V}_1(sC_1) + \hat{V}_O G_2 = 0 \Rightarrow \hat{V}_1 = \frac{-\hat{V}_o G_2}{sC_1}$$

Substituting for $\hat{V}_1$ in the expression for above:

$$\hat{V}_{IN}G_1 + \frac{\hat{V}_O G_1 G_2}{sC_1} + \hat{V}_O sC_2 + \frac{\hat{V}_O G_2 sC_2}{sC_1} + \frac{\hat{V}_O G_2 sC_1}{sC_1} = 0$$

$$\hat{V}_o = \frac{-G_1 \hat{V}_{IN}}{\frac{G_1 G_2}{sC_1} + sC_2 + \frac{G_2 C_2}{C_1} + G_2}$$

Multiplying by s/C_2:

$$\hat{V}_o = \frac{-\frac{G_1 s}{C_2}\hat{V}_{IN}}{s^2 + s(\frac{G_2}{C_1} + \frac{G_2}{C_2}) + \frac{G_1 G_2}{C_1 C_2}}$$

180° Phase Shift →

$$\hat{V}_o = \frac{-s\frac{G_1}{C_2}\hat{V}_{IN}}{s^2 + s(\frac{C_1 + C_2}{C_1 C_2}G_2) + \frac{G_1 G_2}{C_1 C_2}} = 0$$

Characteristic eqn..

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

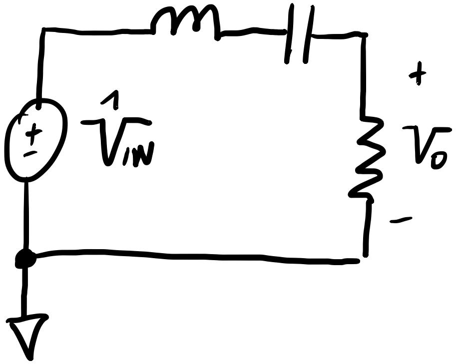
Damping

Natural freq.

$s^2 + 2\alpha s + \omega_0^2 = 0$

$$\hat{V}_o = \frac{-s\frac{G_1}{C_2}\hat{V}_{IN}}{s^2 + s(\frac{C_1+C_2}{C_1C_2}G_2) + \frac{G_1G_2}{C_1C_2}}$$

Comparing the expression here to that from the passive bandpass filter in EE2012 formed by a series RLC combination:



$$\hat{V}_o = \frac{\hat{V}_{IN}\frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$\alpha = \frac{R}{2L}$

Clearly, the two circuits have transfer functions with very similar form except for the $-s$ term in the numerator of the active filter (which introduces a 180° phase shift in the output relative to the input). Othewise, the component values can be chosen to have very similar properties to the passive filter:

.	**Passive BP Filter**	**Active BP Filter**
Characteristic equation	$s^2 + s\frac{R}{L} + \frac{1}{LC}$	$s^2 + s\frac{C_1+C_2}{C_1C_2}G_2 + \frac{G_1G_2}{C_1C_2}$
Natural frequency ω_0	$\sqrt{\frac{1}{LC}}$	$\sqrt{\frac{G_1G_2}{C_1C_2}}$
Damping factor α	$\frac{R}{2L}$	$\frac{C_1+C_2}{2C_1C_2}G_2$

A large number of passive filter configurations can be implemented as active filters, thus simplifying VLSI circuit design variabilities due to inductors. The caveat is that active filters are typically only suitable in low-power on-chip applications.