EE2013

NON-LINEAR CIRCUIT ANALYSIS

LECTURE 19: OPAMP POSITIVE FEEDBACK

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Coordinator: Prof. Pádraig Cantillon-Murphy

LECTURE SCHEDULE

Thursdays 11am-1pm
(with short break)

Monday 9am-10am slot not used!

LECTURE NOTES

https://www.jaeger.ie/ee2013/lec17

Uploaded before lecture takes place

QUESTIONS?

Just ask whenever it comes to you!

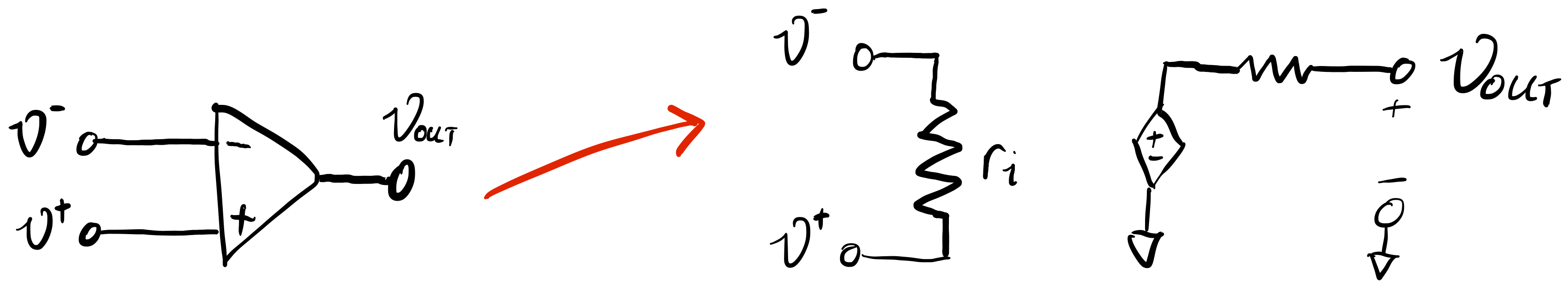
OR:

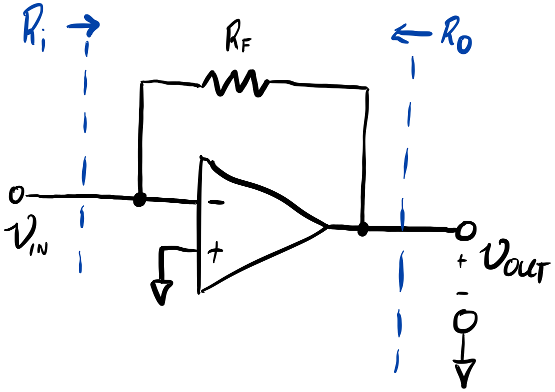
anthony.wall@mcci.ie on Email, Teams or Canvas

1 Review from Last Time

1.1 Op Amp Input and Output Resistance

To analyse the input and output circuit resistance in op amp circuits, we need to add a test source and improve the model of the op amp by adding internal input (r_i) and output (r_o) resistances.





This improved model allows us to calculate the input and output resistance given here for the inverting op amp amplifier:

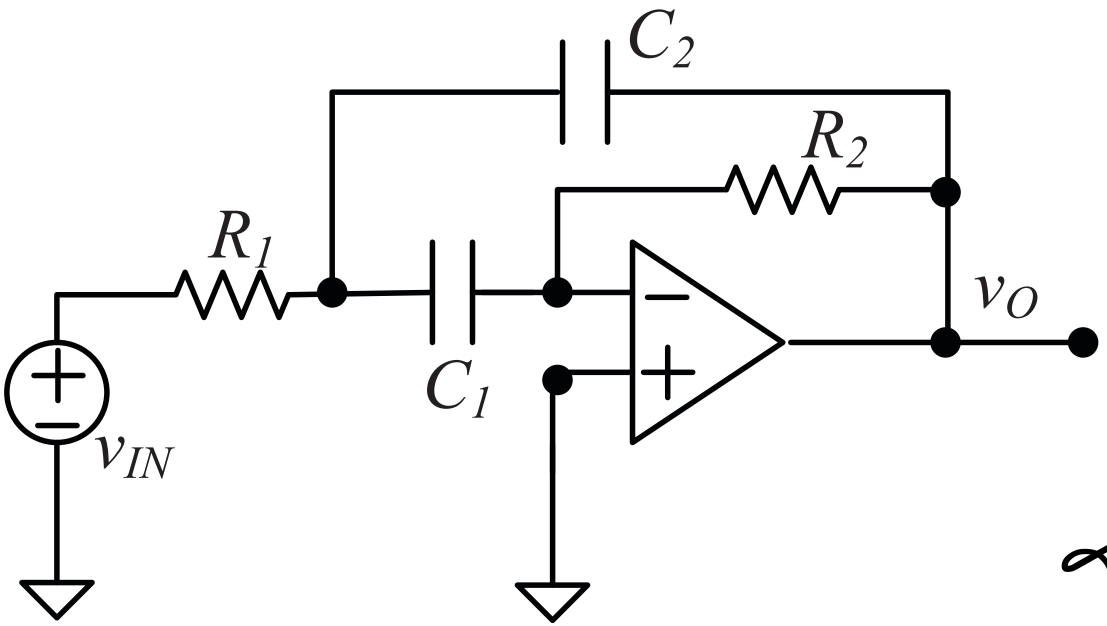
$$R_i = R_S + \frac{R_F + r_o}{A} \approx R_S \quad \text{Dominant}$$

The input resistance should be dominated by the series input resistance.

$$R_o = \frac{r_o}{1 + A\frac{R_S}{R_S+R_F}} \approx \frac{r_o(R_S + R_F)}{AR_S} \quad A >> 1$$

1.2 Active Op Amp Filters

Active op amp filter circuits, like the bandpass filter below, eliminate the need for planar inductors in high-current VLSI circuits.



$$s^2 + 2\alpha s + \omega_0^2 = 0$$

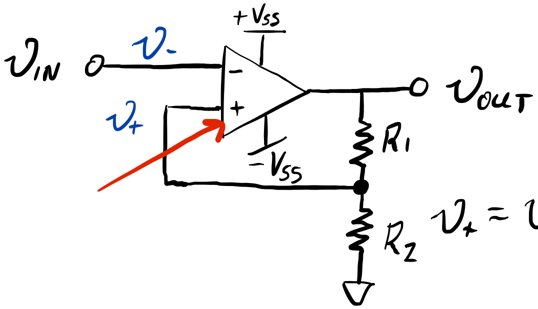
Active Bandpass:

$$\omega_0 = \sqrt{\frac{G_1 G_2}{C_1 C_2}}$$

$$\alpha = \frac{(C_1 + C_2) G_2}{2 C_1 C_2}$$

2 Positive Feedback and Op Amps

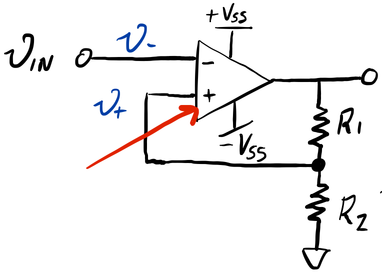
$$v_{OUT} = A(v_+ - v_-)$$



$$v_+ = v_{OUT}\left(\frac{R_2}{R_1+R_2}\right)$$

To date, we have only considered op amp circuits with negative feedback. However, what happens to the non-inverting amplifier circuit if we switch the input terminals to create positive feedback?

In this configuration, the op amp is bistable because once $v^+ - v^-$ becomes slightly positive (negative) the op amp immediately saturates to $+V_S$ ($-V_S$) due to the large positive gain, A. The op amp remains at $+V_S$ ($-V_S$) until $v^+ - v^-$ becomes negative (positive).



$v_+ = v_{OUT}\left(\frac{R_2}{R_1+R_2}\right)$

In the circuit shown, $v^- = v_{IN}$ so the condition for positive saturation assuming that the op amp is initially negatively saturated becomes:

$V_{OUT} = -V_S$

$V_{SS} = V_S$

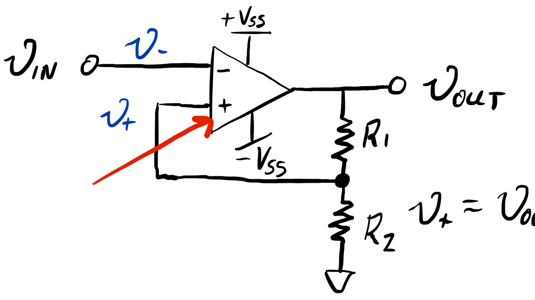
$$A(v^+ - \overset{v_{IN}}{v^-}) = A(-\frac{V_S R_2}{R_1 + R_2} - v_{IN}) > 0$$

$-V_{SS} = -V_S$

For $R_1 = R_2 = R$:

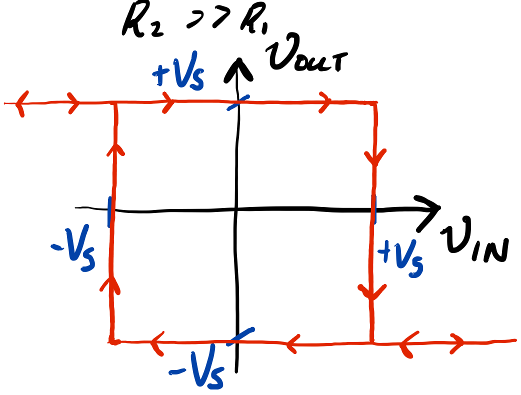
$$A\left(-\frac{V_S}{2} - v_{IN}\right) > 0$$

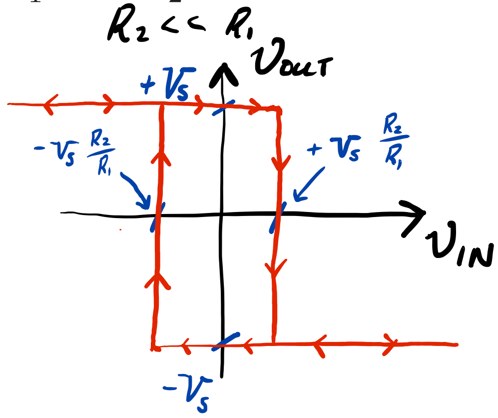
Since A is positive, $v_{IN} < -\frac{V_S}{2}$ changes the output from negative $(-V_S)$ to positive $(+V_S)$ saturation. The output remains in this state until $v_{IN} > \frac{V_S}{2}$, at which point, the output again reverts to $-V_S$.



$$v_+ = v_{OUT}\left(\frac{R_2}{R_1+R_2}\right)$$

The phenomenon by which the output depends on the past input as well as the present input is known as *hysteresis*. Hysteresis results in the loop in the $v_{IN} - v_O$ relation where the width of the hysteresis loop is controlled by R_1 and R_2.

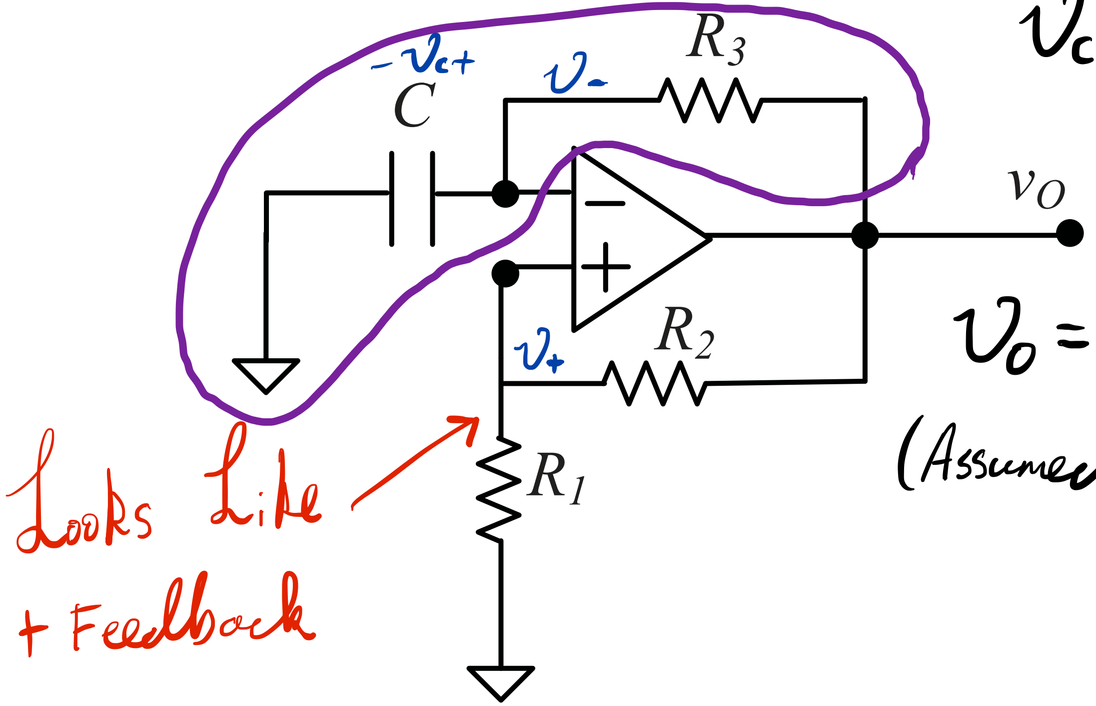




3 The Op Amp Voltage Oscillator

Positive feedback results in op amp saturation, which is useful is some situations. By including RC elements, we can use positive feedback to design a voltage oscillator.

$v_c(t=0) = v_-(t=0) = 0\,V$

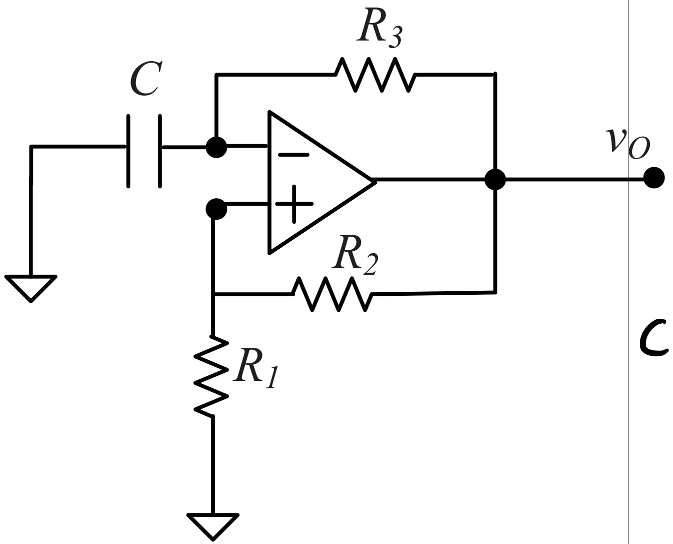


$v_o = A(v_+ - v_-) = V_s$

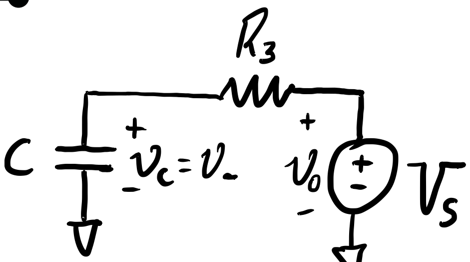
(Assumed State @ $t=0$)

With the capacitor initially in the uncharged state ($v_C = v^- = 0$), the output is positive: $v_O = A(v^+ - v^-) = V_S$.

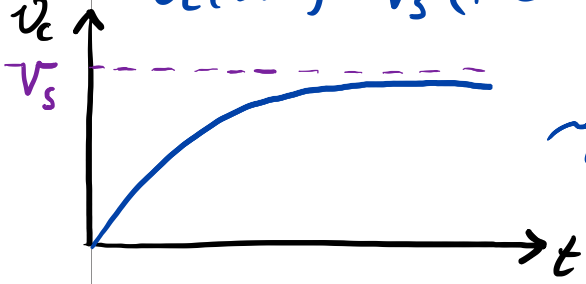
$$v_+ = v_o \frac{R_1}{R_1 + R_2} = V_s \frac{R_1}{R_1 + R_2}$$



However, since the v^- terminal represents a large input impedance, the output voltage causes the capacitor to begin charging.

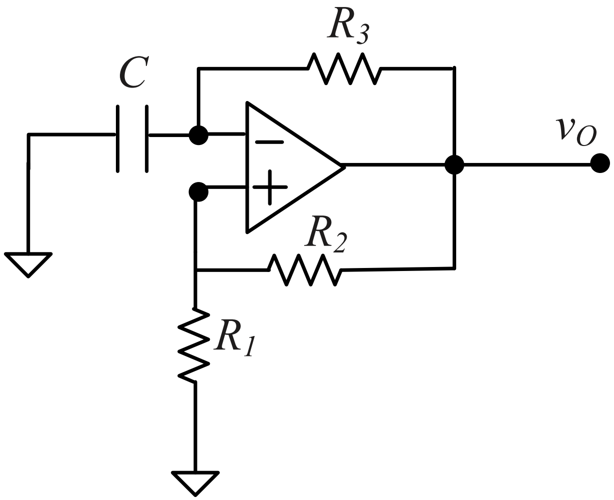


$$v_c(t>0) = V_s\left(1-e^{-t/\tau_1}\right)$$



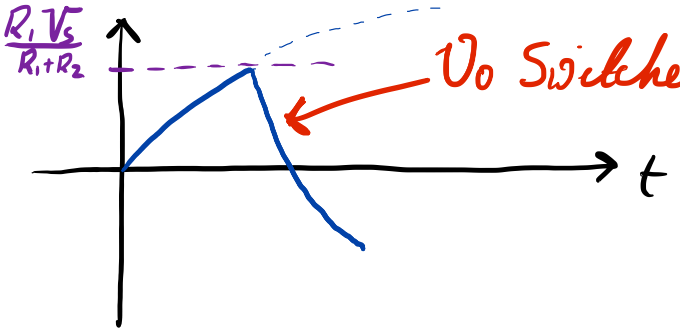
$$\tau_1 = R_3 C$$

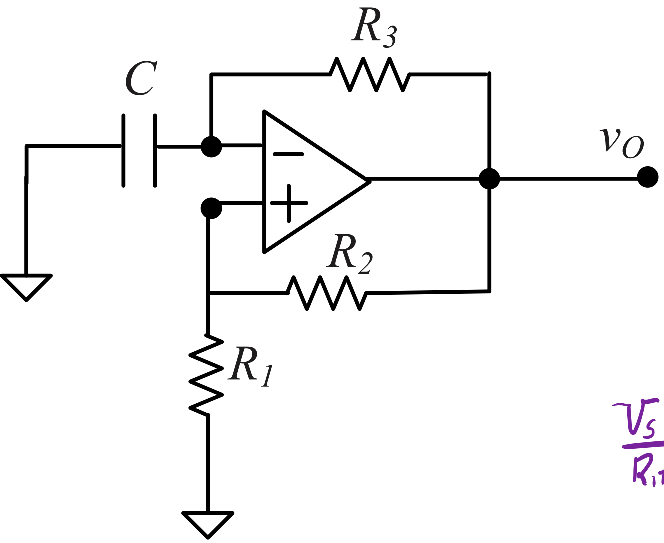
As the capacitor charges, $v_C = v^-$ will eventually exceed $v^+ = \frac{R_1 V_S}{R_1+R_2}$ resulting in the output, $v_O = A(v^+ - v^-)$, switching from positive ($+V_S$) to negative ($-V_S$). This happens when:



$$v_c = v^- > v^+$$

$$V_S\left(1-e^{-t/\tau_1}\right) > \frac{R_1 V_S}{R_1+R_2}$$

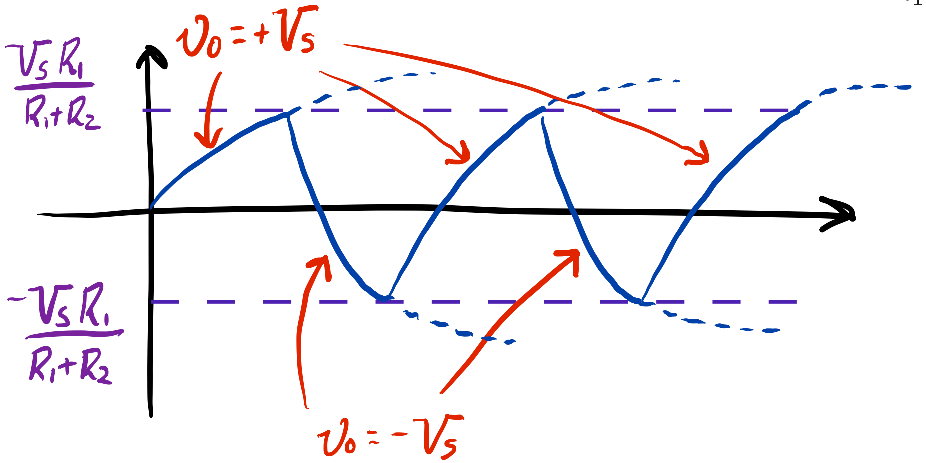




Once the output changes to $-V_S$, the positive input, v^+, also changes:

$$v^+ = \frac{R_1 v_O}{R_1 + R_2} = -\frac{R_1 V_S}{R_1 + R_2}$$

Now, the partially charged capacitor has a higher voltage than the output voltage and begins to discharge through R_3. The capacitor continues to discharge until $v_C = v^- = -\frac{V_S R_1}{R_1+R_2}$



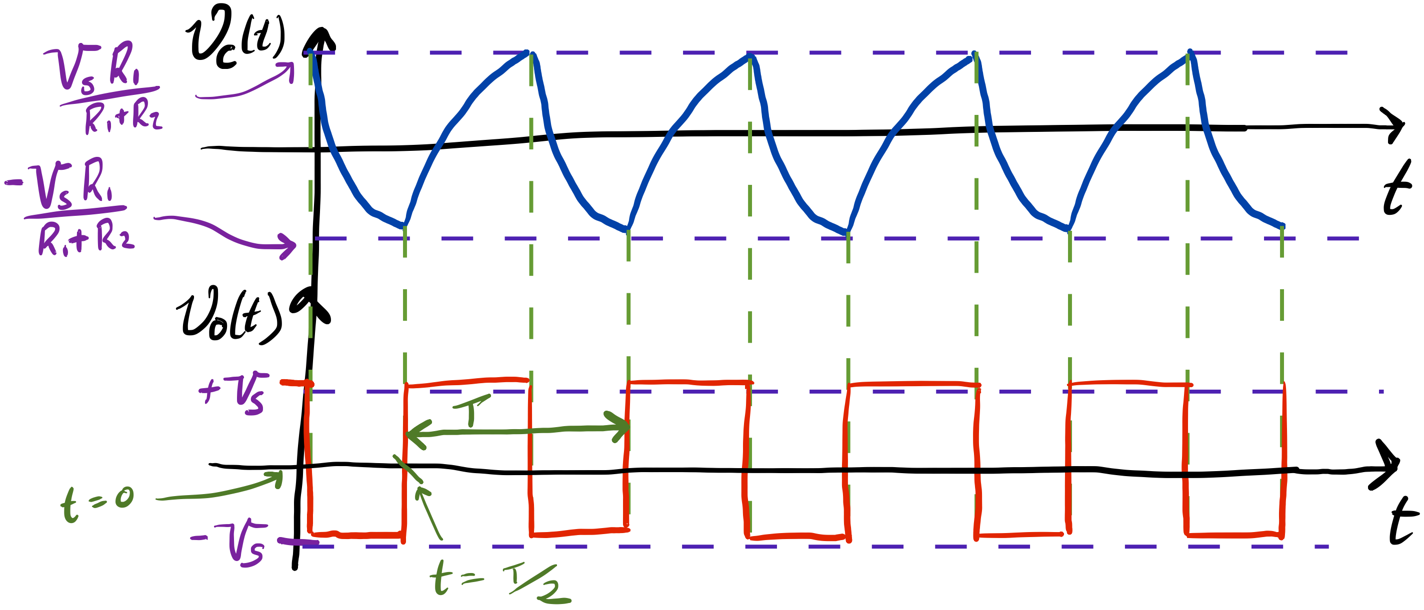
As soon as the capacitor voltage then exceeds $-\frac{R_1 V_S}{R_1+R_2}$, this causes $A(v^+ - v^-)$ to turn positive and the op amp returns to positive saturation:

$$v_o = +V_s$$

$$v_+ = \frac{R_1 V_s}{R_1+R_2}$$

The capacitor then begins charging again and so on, the process repeating itself everytime that the capacitor voltage satisfies the condition:

$$v_c(t) = \pm \frac{V_s R_1}{R_1+R_2}$$



3.1 Oscillator Time Constants

An important parameter is the oscillation period, T, which depends on the component values. To determine T, consider the discharging cycle of the capacitor:

$$V_C(t=0^-) = V_C(t=0^+) = \frac{V_S R_1}{R_1+R_2}$$

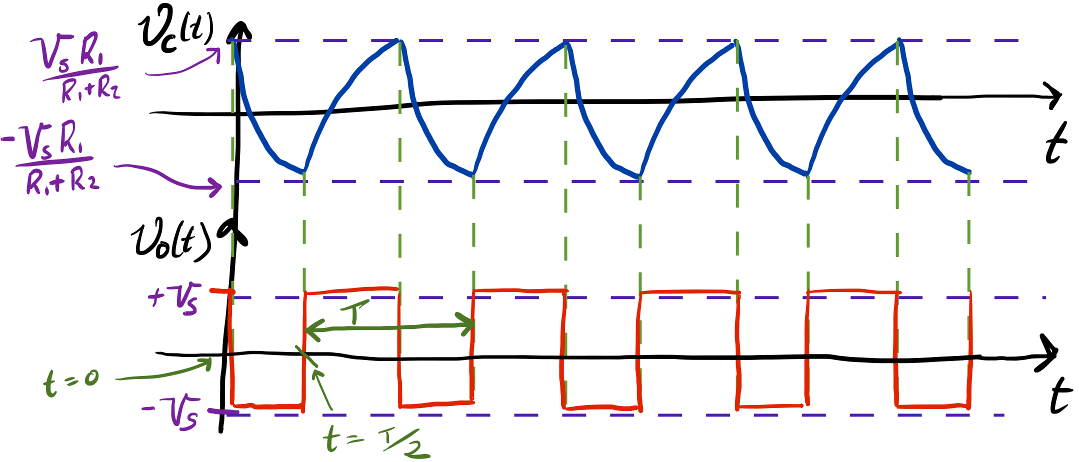
Since $v_O = -V_S$ at $t = 0^-$, the final possible capacitor voltage at $v_C(t \gg \tau_1)$ is $-V_S$. In general:

$$v_C(t) = \overbrace{v_C(0^+)e^{-t/\tau}}^{\text{Initial}} + \overbrace{v_C(t \gg \tau)(1 - e^{-t/\tau})}^{\text{Final Value} \to -V_S}$$

$$\Rightarrow v_C(0 < t < T/2) = \frac{V_S R_1}{R_1+R_2} e^{-t/\tau} + (-V_S)\left(1 - e^{-t/\tau_1}\right)$$

The output voltage switches from $-V_S$ to $+V_S$ at $t = T/2$, at which point:

$$v_C(t = T/2) = \frac{R_1 \cancel{V_S}}{R_1 + R_2}\, e^{-T/(2\tau_1)} - \cancel{V_S} + \cancel{V_S}\, e^{-T/(2\tau_1)} = \frac{-\cancel{V_S} R_1}{R_1+R_2}$$



$$e^{-T/2\tau_1}\left(\frac{R_1}{R_1+R_2} + 1\right) = 1 - \frac{R_1}{R_1+R_2}$$

$$\frac{R_1+R_2}{R_1+R_2}$$

$$e^{-T/2\tau_1}\left(\frac{R_1+R_2+R_1}{\cancel{R_1+R_2}}\right) = \frac{R_2}{\cancel{R_1+R_2}}$$

$$e^{-T/(2\tau_1)} = \frac{R_2}{2R_1 + R_2}$$

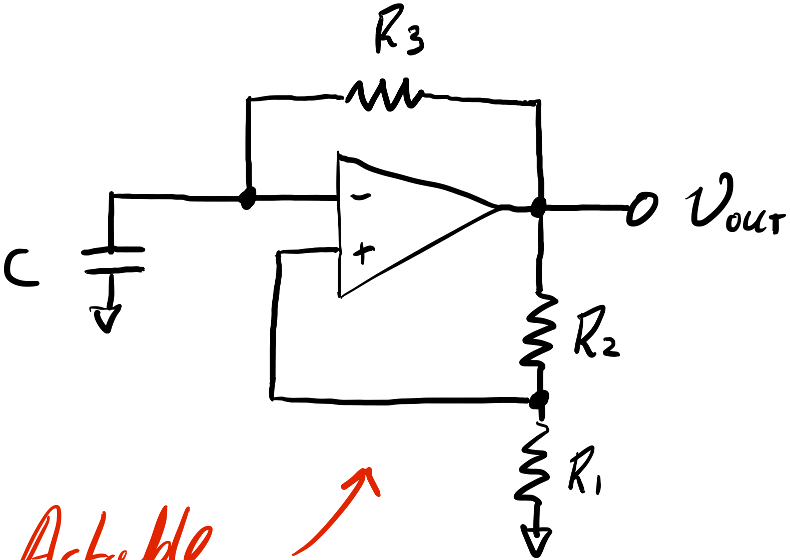
$$-\frac{T}{2\tau_1} = \ln\left(\frac{R_2}{2R_1+R_2}\right)$$

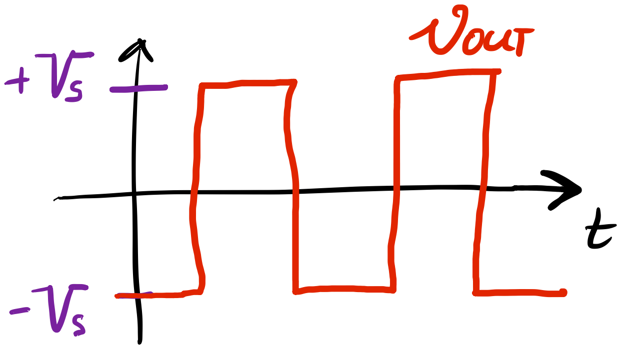
$$\frac{T}{2\tau_1} = ln(\frac{2R_1 + R_2}{R_2}) = ln(\frac{2R_1}{R_2} + 1)$$

$$T = 2\tau_1 \ln\left(\frac{2R_1}{R_2} + 1\right)$$

$$T = 2R_3C \; ln(\frac{2R_1}{R_2} + 1)$$

This is the simplest op amp oscillator design and is the basis for the 555 timer IC.





Astable Multivibrator