EE2013

NON-LINEAR CIRCUIT ANALYSIS

LECTURE 20: OPAMP FILTER DESIGN 2

Instructors: Alex Jaeger, Anthony Wall
Coordinator: Prof. Pádraig Cantillon-Murphy

LECTURE SCHEDULE

Thursdays 11am-1pm
(with short break)

Monday 9am-10am slot not used!

LECTURE NOTES

https://www.jaeger.ie/ee2013/lec17

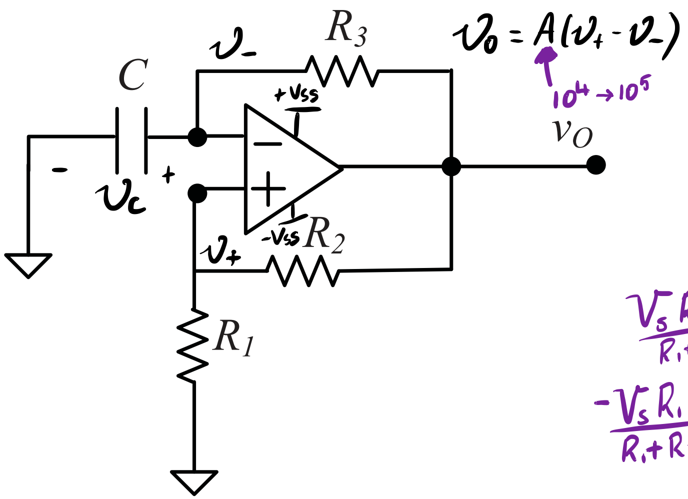
Uploaded before lecture takes place

QUESTIONS?

Just ask whenever it comes to you!

OR:

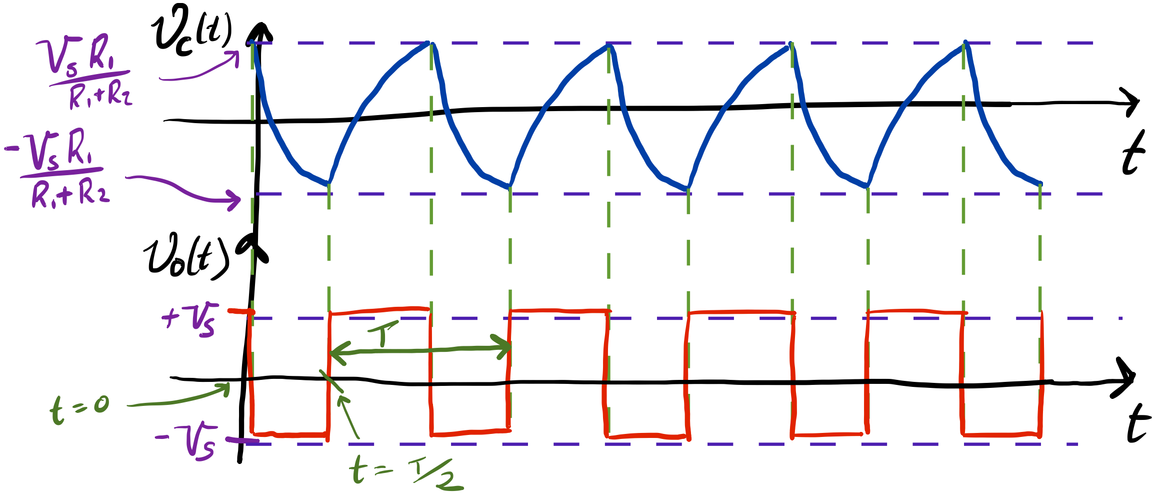
anthony.wall@mcci.ie on Email, Teams or Canvas



1 Review from Last Time

1.1 Op Amps in Positive Feedback

Positive feedback results in op amp saturation, which is useful in some situations such as the voltage oscillator circuit.



The resultant period of oscillation from this circuit is given by:

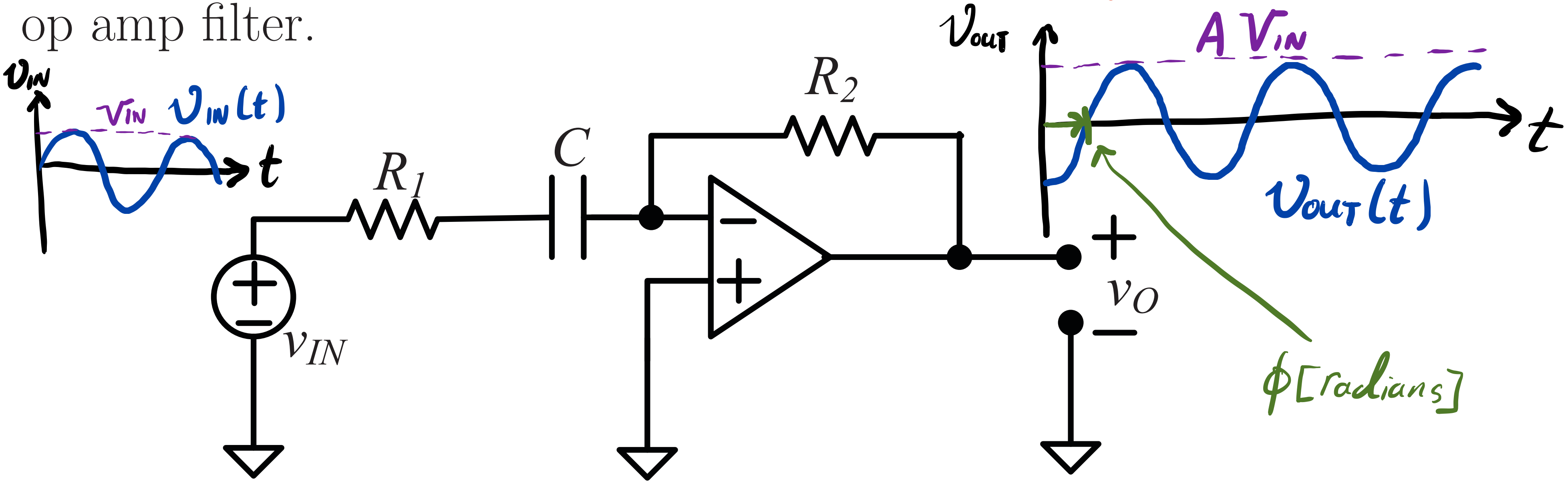
$$T = 2R_3Cln(\frac{2R_1}{R_2} + 1) \qquad \tau_1 = R_3C$$

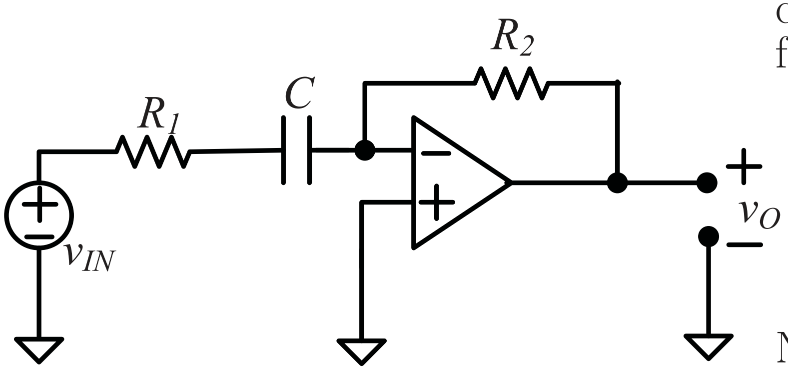
2 Active Filter Frequency Response

Today we return to the frequency response characteristics of op amps with applications to filtering. We have already seen in EE2012 how passive (RLC) components can be used to create filters. However, we have now seen that filtering can also be achieved with op-amp circuits which can increase the gain of small passive components. This is called *active filtering*. We will now look to graphically interpret op amp frequency response as we did for passive (RLC) filters.

2.1 Op Amp High Pass Filter

The circuit below can be used to implement a ~~low~~ high pass first-order op amp filter.





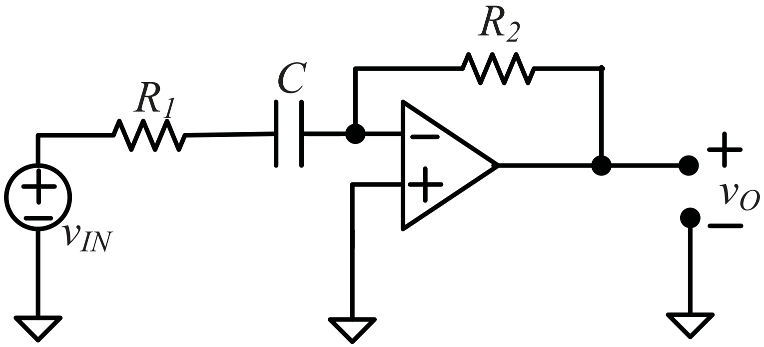
To analyse the circuit in sinusoidal steady state, we use complex coefficients for the signals (voltages and currents) and impedances for the passive components.

$\hat{V}_i = V_i$

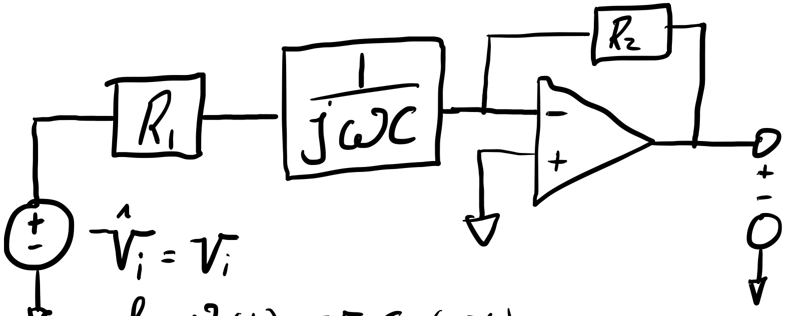
$$v_{IN} = V_i \cos(\omega t) = \Re e\{\hat{V}_i e^{\jmath\omega t}\} = \Re e\{V_i e^{\jmath\omega t}\}$$

$$v_O = V_o \cos(\omega t + \phi) = \Re e\{\hat{V}_o e^{\jmath\omega t}\} = \Re e\{A V_i e^{\jmath\phi} e^{\jmath\omega t}\}$$

Note that ϕ is the phase shift in radians at the output and for the example here, $\hat{V}_i = V_i$.



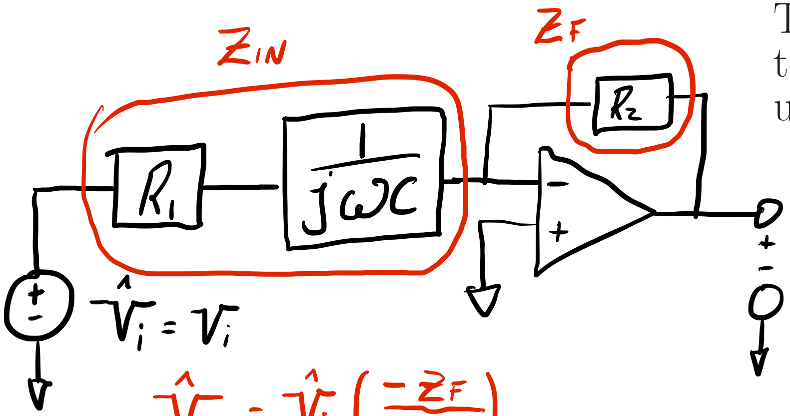
$j = \sqrt{-1}$



$\hat{V}_i = V_i$

for $v_i(t) = V_i \cos(\omega t)$

$\hat{V}_o = A V_i e^{j\phi}$

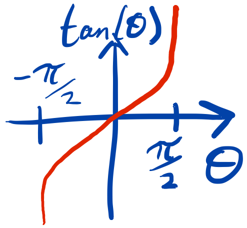


$$\hat{V}_o = \hat{V}_i\left(\frac{-Z_F}{Z_{IN}}\right)$$

The complex coefficient for the output voltage can be written in terms of the complex coefficient for the input voltage using the usual relationship for the inverting amplifier.

$$\frac{\hat{V}_o}{\hat{V}_i} = \frac{-R_2}{R_1 + \frac{1}{j\omega C}} = \frac{-j\omega R_2 C}{j\omega R_1 C + 1}$$

$$\frac{\hat{V}_o}{\hat{V}_i} = \frac{\omega R_2 C}{\sqrt{1+\omega^2 R_1^2 C^2}} e^{\jmath(-\frac{\pi}{2} - \arctan(\omega R_1 C))}$$



The resultant time domain expression for the output voltage is determined from the real part of the complex expressions as usual:

$$V_o(t) = \frac{\omega R_2 C V_i}{\sqrt{1+\omega^2 R_1^2 C^2}} \cos\left(\omega t - \frac{\pi}{2} - \arctan(\omega R_1 C)\right)$$

$$A = \frac{V_o}{V_i} = \frac{\omega R_2}{\sqrt{1+\omega^2 R_1^2 C^2}}$$

$$\phi = \angle V_o - \angle V_i = -\frac{\pi}{2} - \operatorname{atan}(\omega R_1 C)$$

3 Bode Plots for Op Amp Circuits

Let's look at the resulting magnitude and phase of the output voltage for different frequencies, ω, referenced to the input.

3.0.1 Magnitude Bode Plot

In plotting the frequency response in a Bode plot, it is usual to use a log scale for both frequency, ω, and the ratio of magnitudes, A.

$$A = \frac{\left|\hat{V}_o\right|}{\left|\hat{V}_i\right|} = \frac{V_o}{V_i}$$

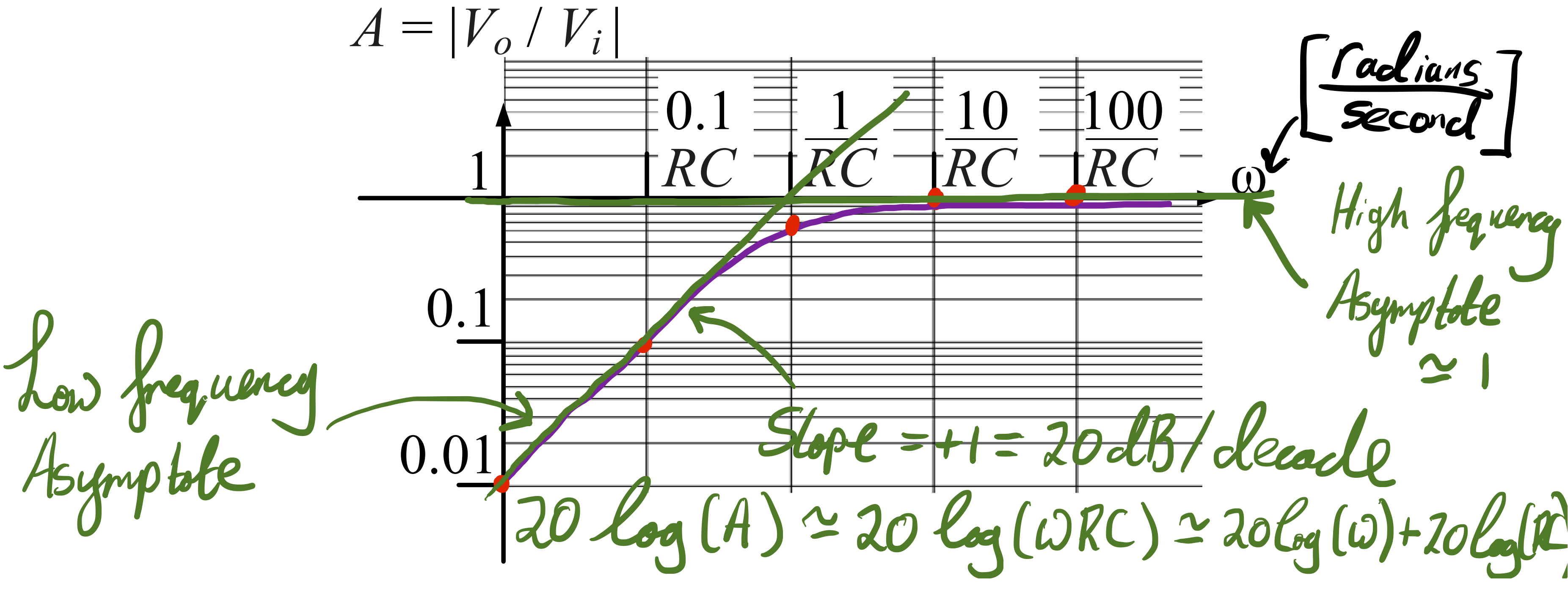
Consider the limiting cases for the case when $R_1 = R_2 = R$:

$$A = \frac{V_o}{V_i} = \frac{\omega R_2 C}{\sqrt{1+\omega^2 R_1^2 C_2}}$$

$$\omega \ll \frac{1}{RC} \quad \Rightarrow A \simeq \omega R_2 C = \omega RC$$

$$\omega \gg \frac{1}{RC} \quad \Rightarrow A \simeq \frac{\omega R_2 C}{\sqrt{\omega^2 R^2 C^2}} = 1$$

$$\omega = \frac{1}{RC} \quad \Rightarrow A \simeq \frac{\left(\frac{1}{RC}\right) R_2 C}{\sqrt{1+\left(\frac{1}{RC}\right)^2 R_1^2 C^2}} = \frac{1}{\sqrt{2}} \simeq 0.707$$



3.1 Phase Bode Plot

The phase Bode plot is the difference in phase between the input and output voltages. Again, frequency is plotted on a log scale but it's generally more common to plot the phase difference on a linear scale.

$$\measuredangle V_o = -\frac{\pi}{2} - \operatorname{atan}(\omega RC)$$

$$\measuredangle V_i = 0$$

$$\phi = -\frac{\pi}{2} - \operatorname{atan}(\omega RC)$$

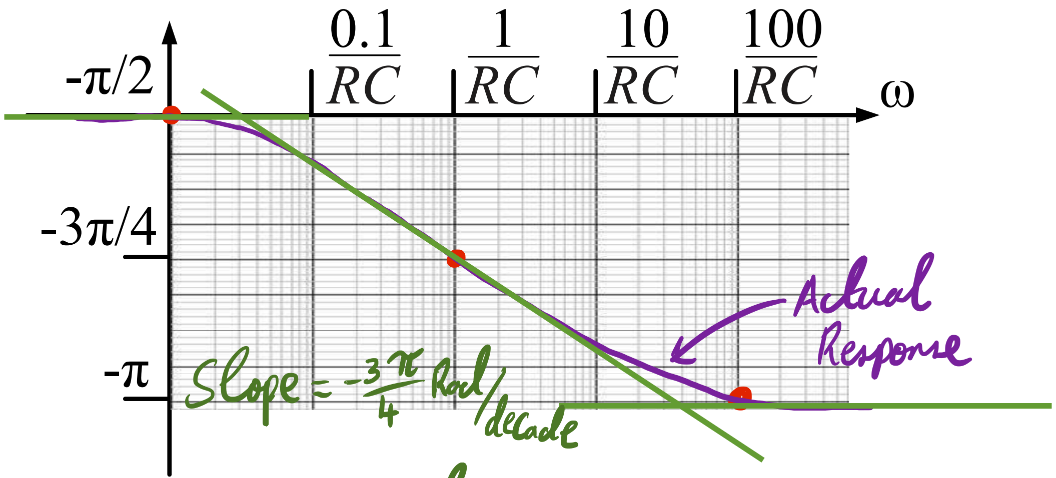
Consider the limiting cases, again when $R_1 = R_2 = R$:

$$\omega \ll \frac{1}{RC} \quad \Rightarrow \phi \simeq -\frac{\pi}{2} - \operatorname{atan}(0) = -\frac{\pi}{2}$$

$$\omega \gg \frac{1}{RC} \quad \Rightarrow \phi \simeq -\frac{\pi}{2} - \operatorname{atan}(\infty) = -\frac{\pi}{2} - \frac{\pi}{2} = -\pi$$

$$\omega = \frac{1}{RC} \quad \Rightarrow \phi = -\frac{\pi}{2} - \operatorname{atan}(1) = -\frac{\pi}{2} - \frac{\pi}{4} = -\frac{3\pi}{4}$$

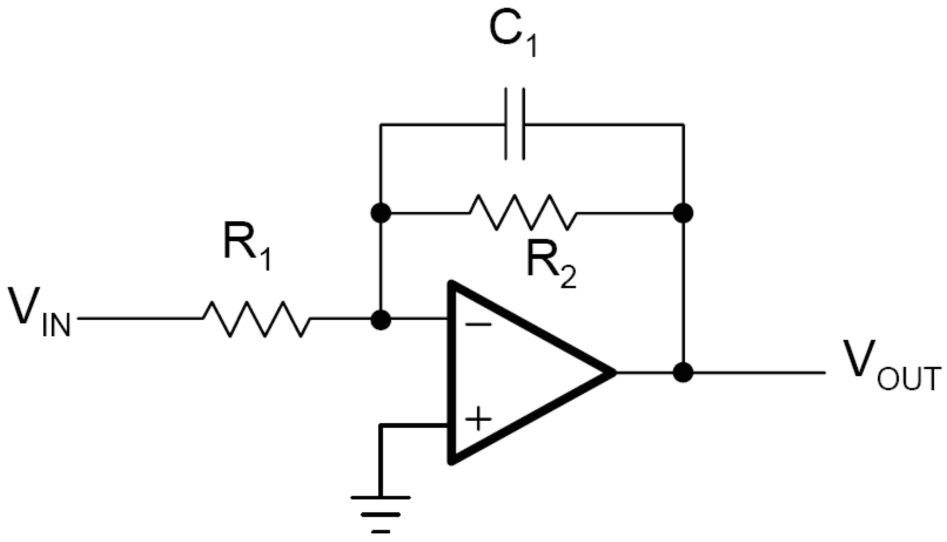
$\phi = \angle v_O - \angle v_{IN}$



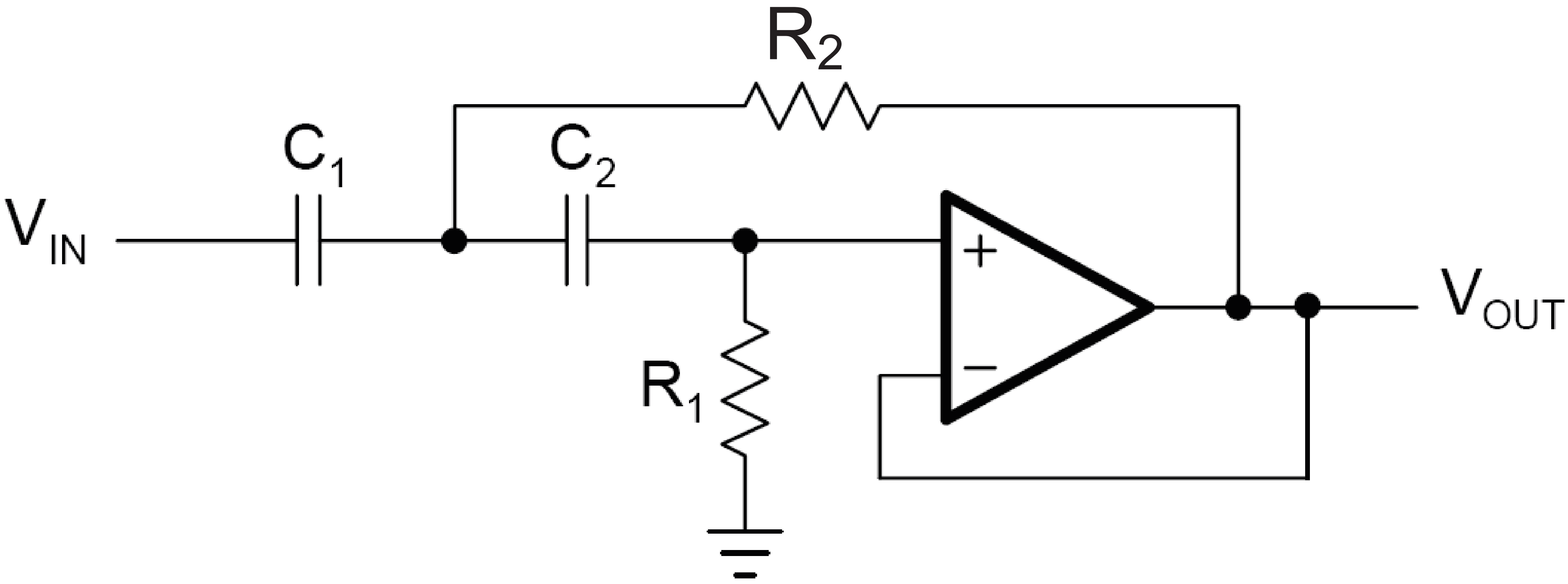
$= -45$ deg/decade

4 Op Amp Filter Topologies

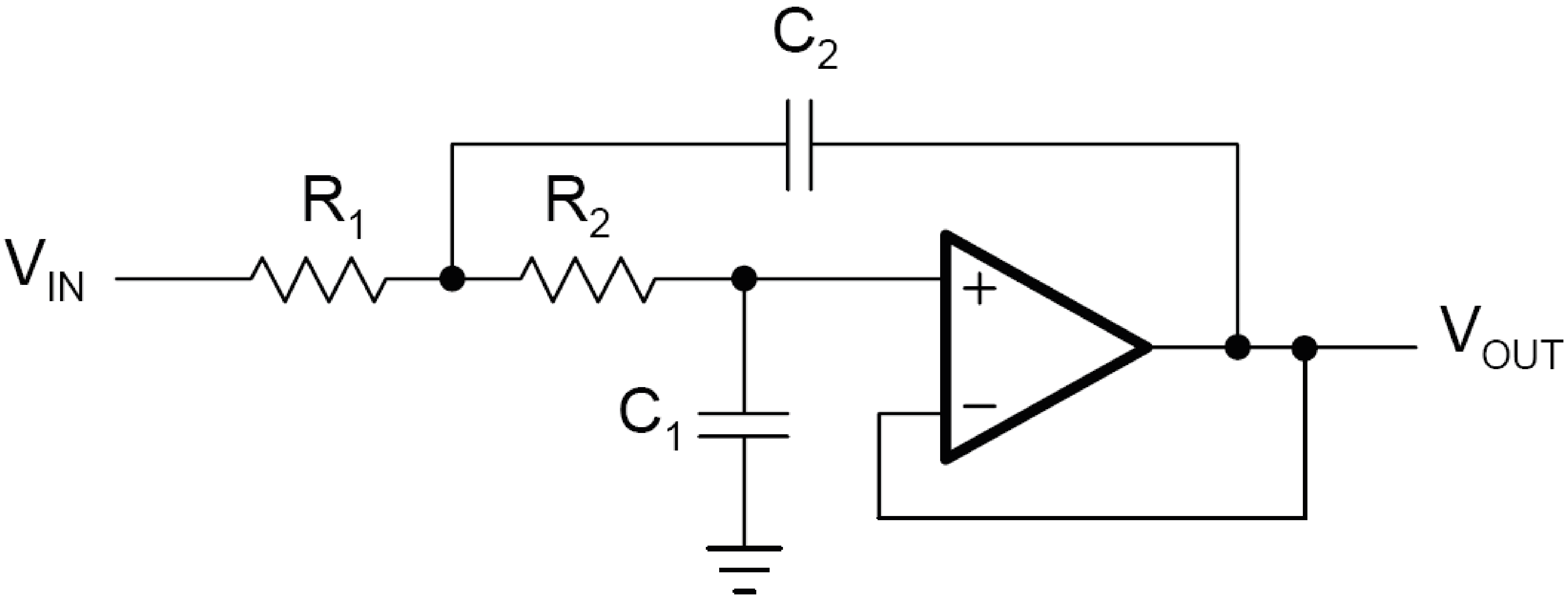
Op amps can be configured in a wide variety of topologies which changes the frequency response of the output voltage. A complete guide to op amp filter design is available online at *http://www.cypress.com/file/65366/download*



First order low pass filter



Second order high pass filter

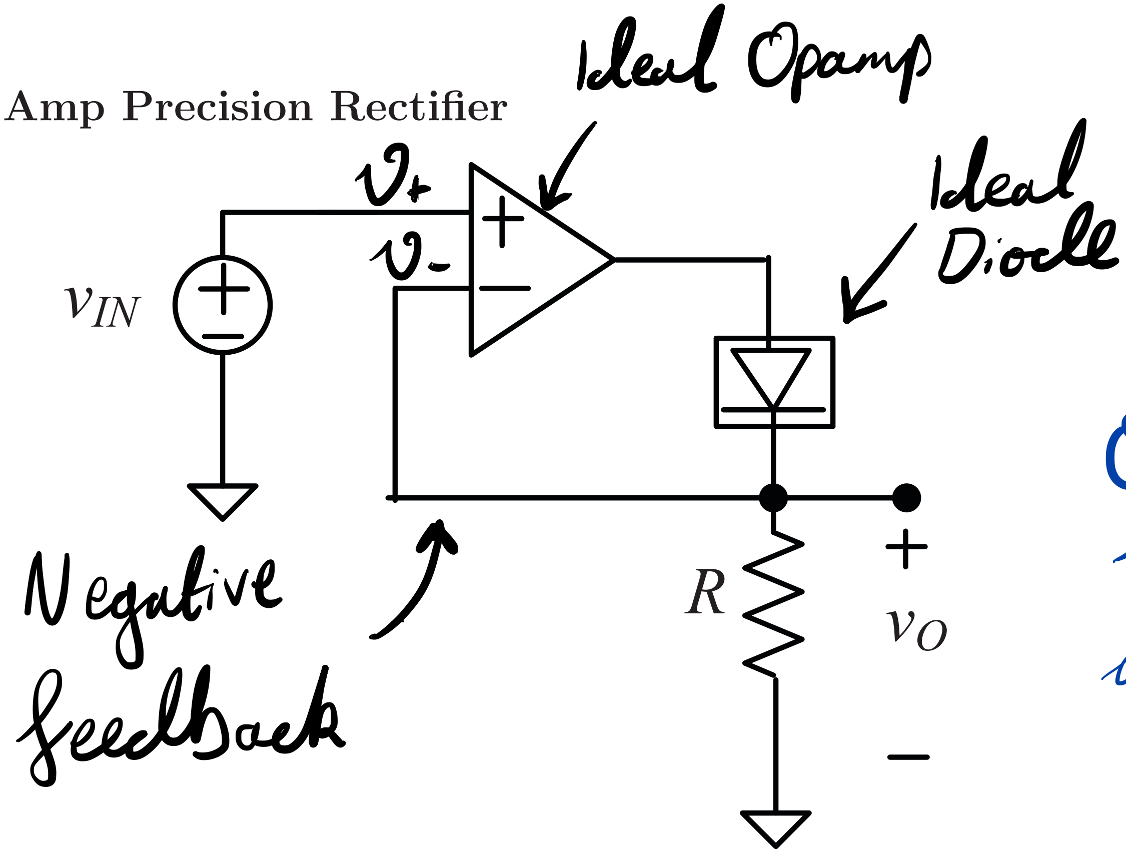


Second order low pass filter (Sallen Key)

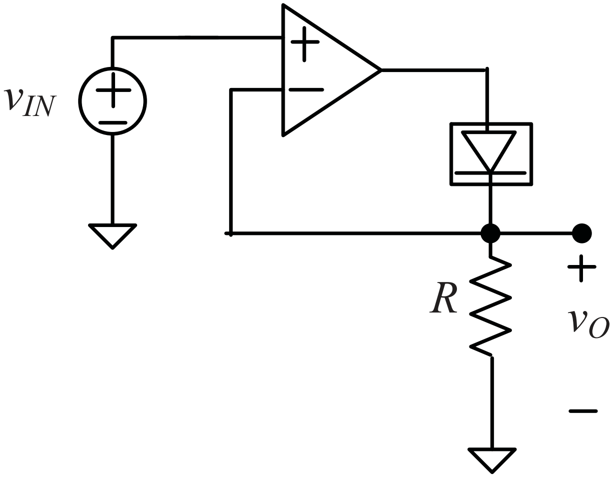
5 Op Amp Diode Circuits

By combining op amps with diodes, a number of useful circuits result. One example of this is the op amp precision rectifier.

5.1 Op Amp Precision Rectifier



OpAmp acts to keep $v_- \simeq v_+$ in negative feedback

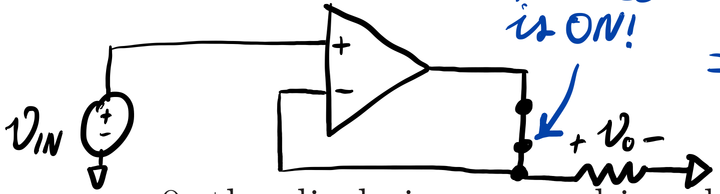


When $v_{IN} > 0$, the diode is forward biased by the positive amplifier gain, A, and the circuit behaves like a buffer.

Diode is ON!

$v_{IN} = v_+ \simeq v_- = v_o$

$\Rightarrow v_o \simeq v_{IN}$



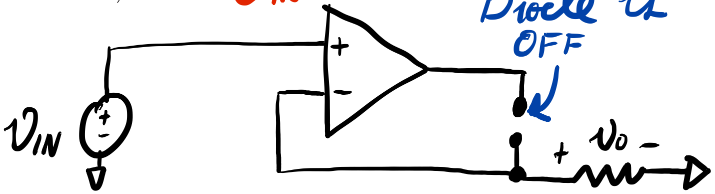
When $v_{IN} < 0$, the diode is reverse biased and no current flows in the resistor, R. $v_{IN} < 0$

Diode is OFF

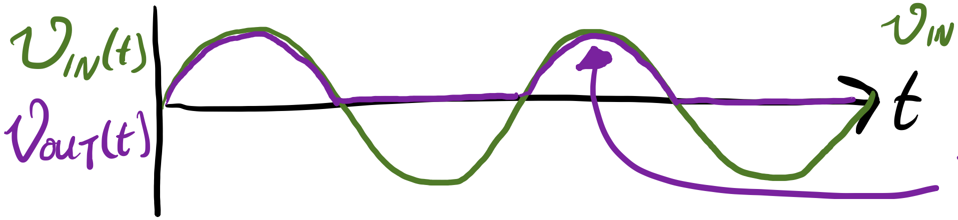
OpAmp has no Control over v_-

$v_- \neq v_+$

$v_o \simeq 0$



The result is the precision rectifier (sometimes called a *super-diode*), which finds application in high-precision signal processing.

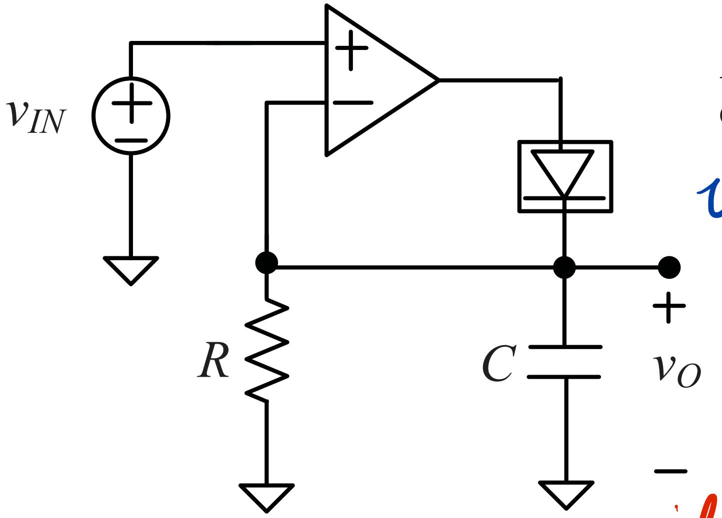


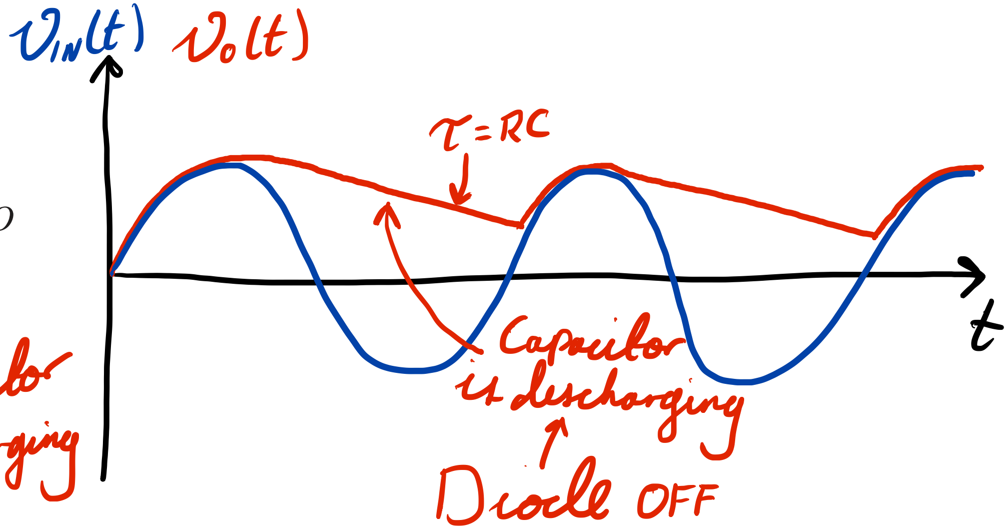
$v_{IN}(t) = V\sin(\omega t)$

No forward Voltage drop!

5.2 Op Amp Peak Detector

Adding an output capacitor to the precision rectifier results in the op amp peak detector.





$$v_{IN}(t) = V\sin(\omega t)$$

Capacitor is Charging

Diode ON

If the RC time constant is chosen appropriately for the input signal period, the output, v_O, can respond to each positive periodic input peak in much the same way as a passive demodulator circuit does but with ~~a buffered output~~. No forward Voltage drop!