EE2013

NON-LINEAR CIRCUIT ANALYSIS

LECTURE 21: INTERESTING OPAMP CIRCUITS

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Coordinator: Prof. Pádraig Cantillon-Murphy

LECTURE SCHEDULE

Last Lecture!

Tutorial Session @ 12 - Anthony's Material

Alex will record some video lectures on his material

LECTURE NOTES

https://www.jaeger.ie/ee2013/lec21
Uploaded before lecture takes place

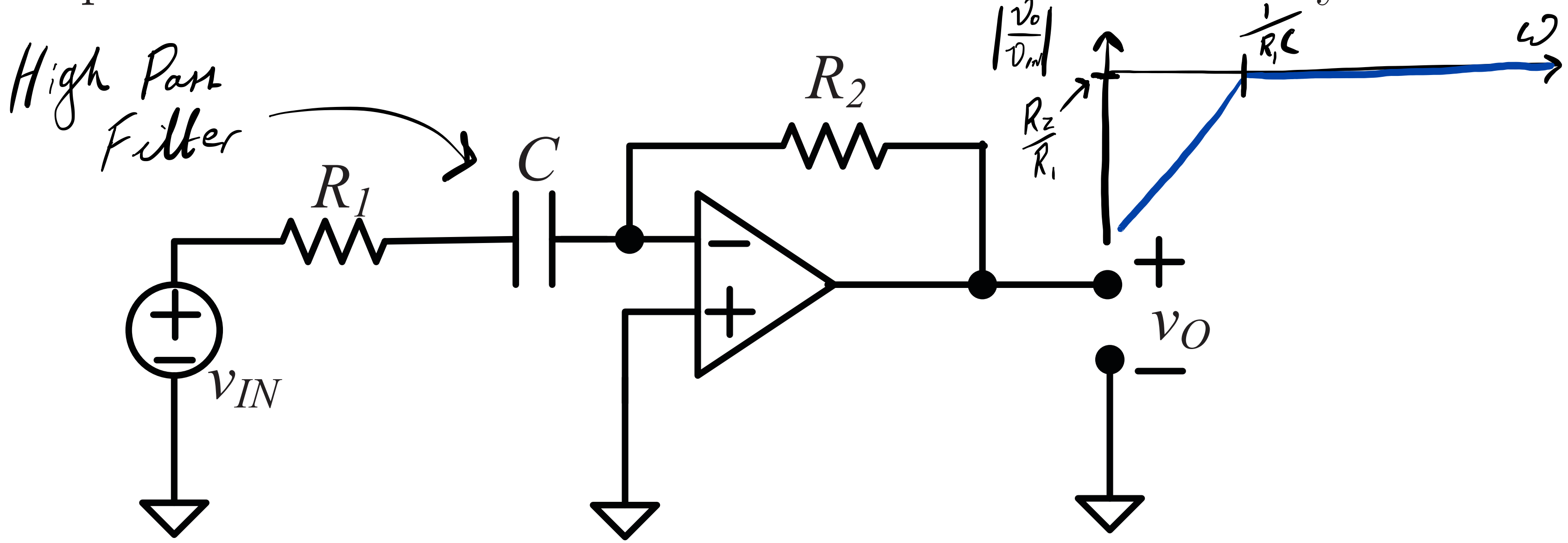
QUESTIONS?

Just ask whenever it comes to you!
OR:
anthony.wall@mcci.ie on Email, Teams or Canvas

1 Review from Last Time

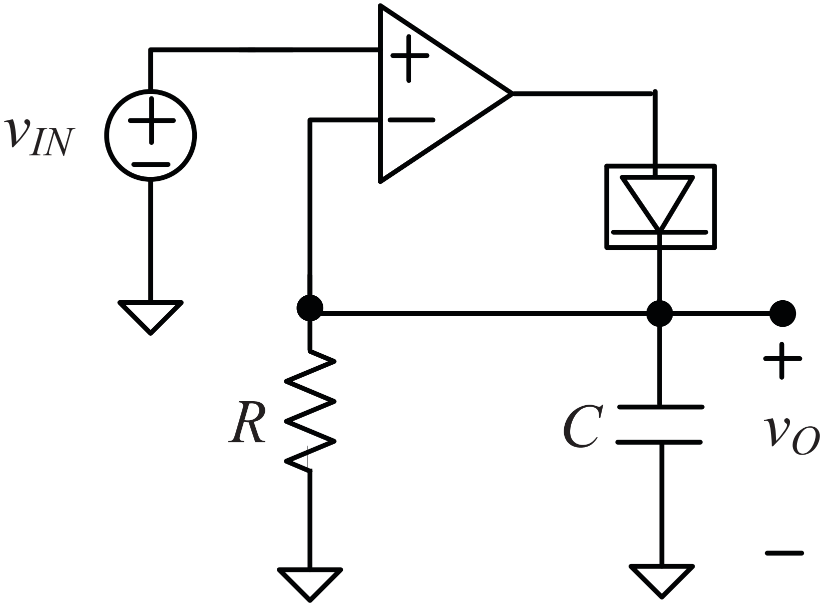
1.1 Op Amps for Active Filtering

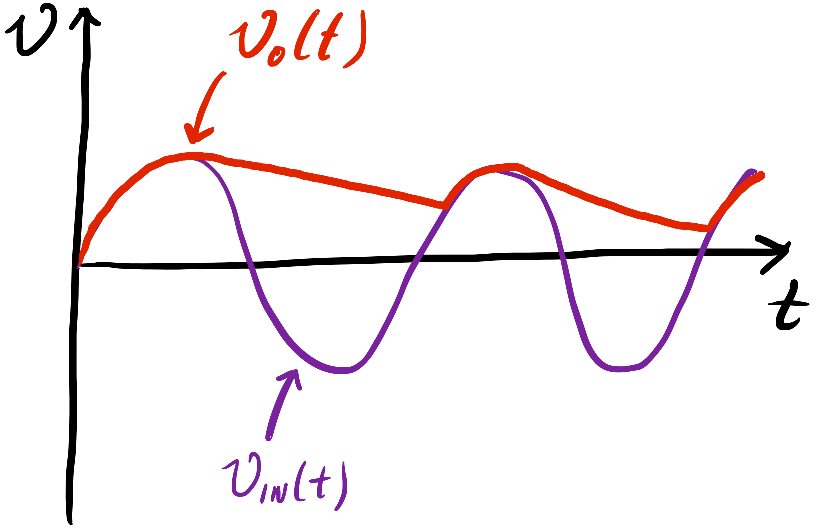
Op amps can be used as active filters in sinusoidal steady state.



1.2 Op Amp Diode Circuits

The precision rectifier and peak detector can be implemented with op amp circuits. Op amp implementation has the advantage of presenting a high-impedance input to preceding stages.

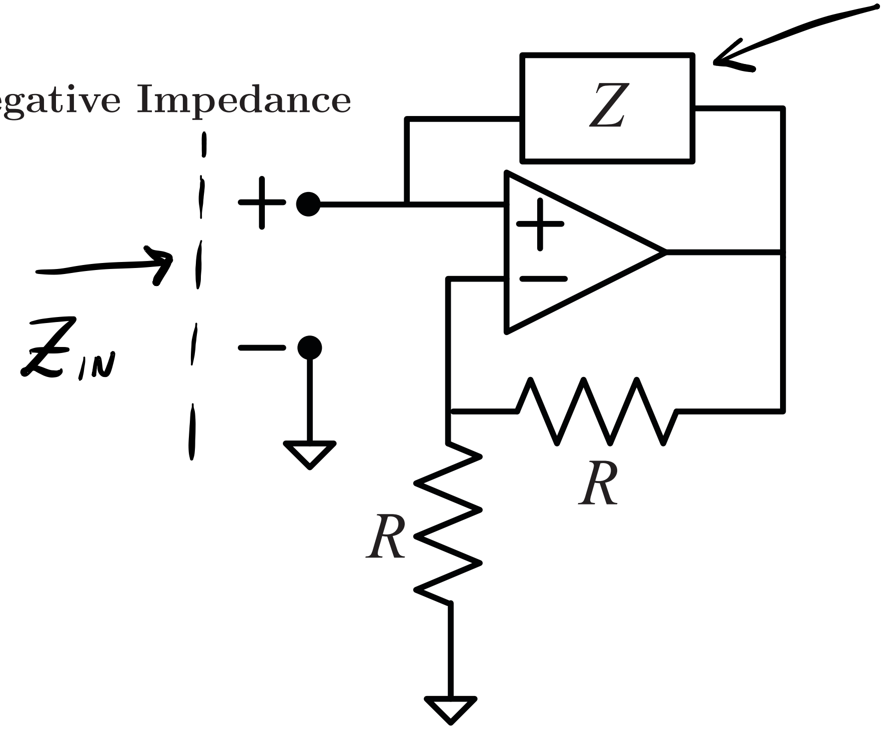




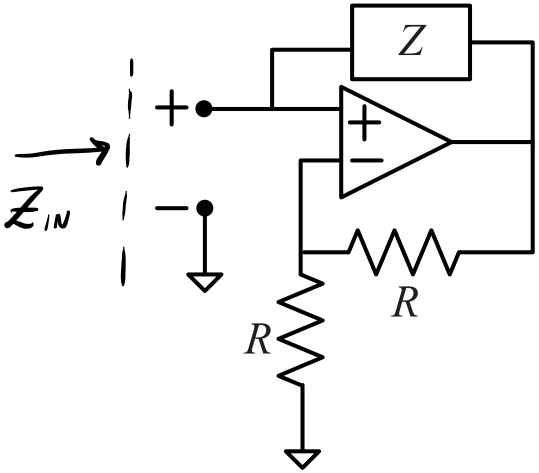
2 Interesting Op Amp Circuits

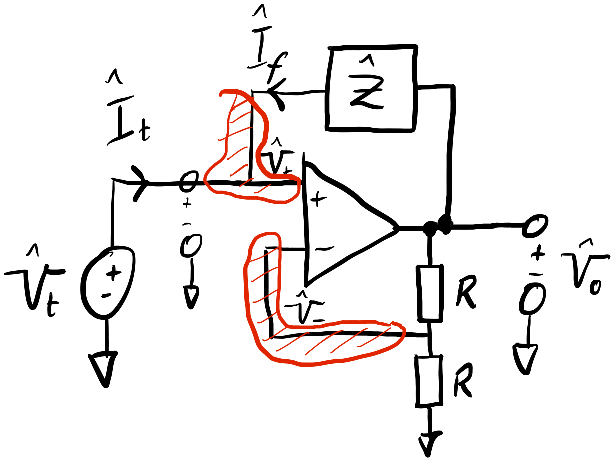
Today, we are going to look at some of the more interesting applications of op amps. The first of these is the use of an op amp to implement negative impedance.

2.1 Creating Negative Impedance

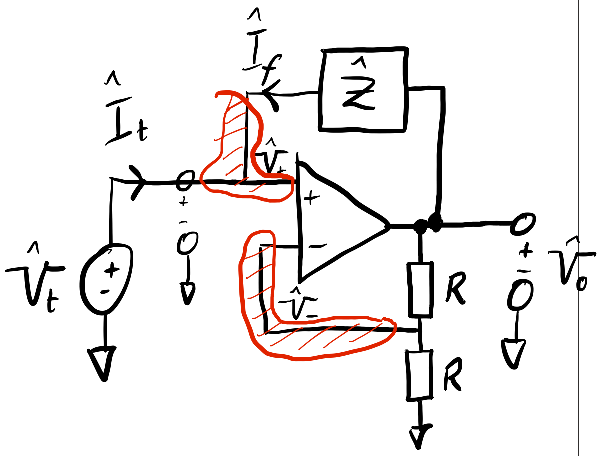


We are interested in determining the input impedance under sinusoidal steady state conditions seen from the input terminals in the circuit above. To do this, we will use complex coefficients to represent the voltages and currents, and complex impedances to represent the passive elements. Then, we apply a test voltage source at the input terminals. The general impedance, Z, might be a capacitor, an inductor or a resistor.





$$v_t = V_t \cos(\omega t) = \mathrm{Re}\{\hat{V}_t e^{j\omega t}\}$$



Applyng KCL at the non-inverting op amp input:

$$\sum i_{IN} - \sum i_{OUT} = \hat{I}_t + \hat{I}_f = \hat{I}_t + \left(\frac{\hat{V}_o - \hat{V}_+}{\hat{Z}}\right) = 0$$

Negative feedback $V_+ \simeq V_-$

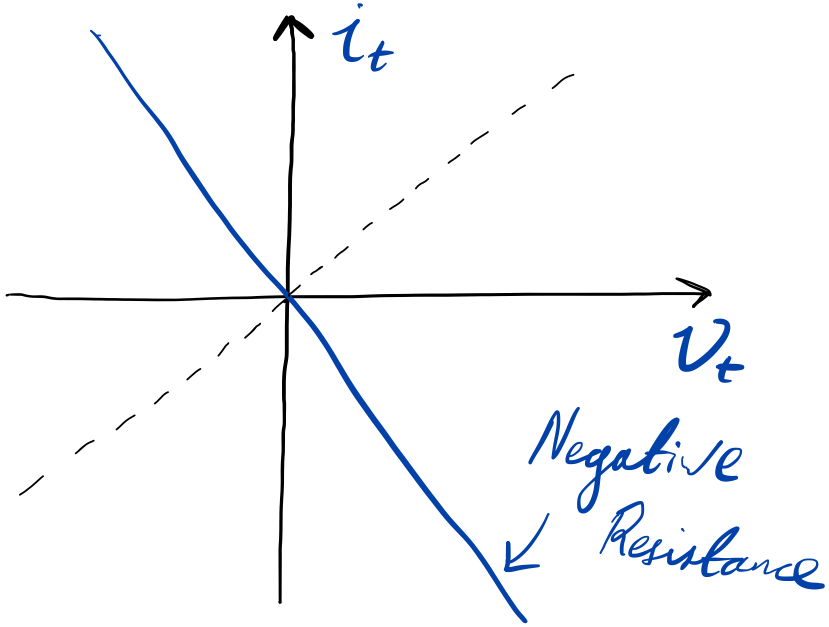
$$\hat{V}_O = 2\hat{V}^- = 2\hat{V}^+ = 2\hat{V}_t$$

$$\hat{I}_t + \frac{2\hat{V}_t}{\hat{Z}} - \frac{\hat{V}_t}{Z} = \hat{I}_t + \frac{\hat{V}_t}{\hat{Z}} = 0$$

The circuit converts the impedance Z to its negative. For example, negative resistors were used to cancel resistance in telephone cables, serving as repeaters.

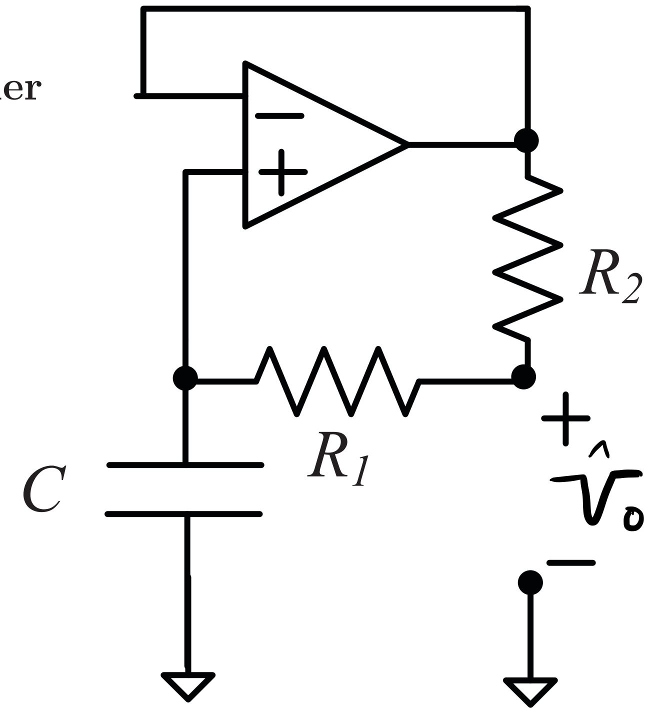
$$\frac{\hat{V}_t}{\hat{I}_t} = Z_{IN} = -Z$$

$$\frac{v_t}{i_t} = -Z$$

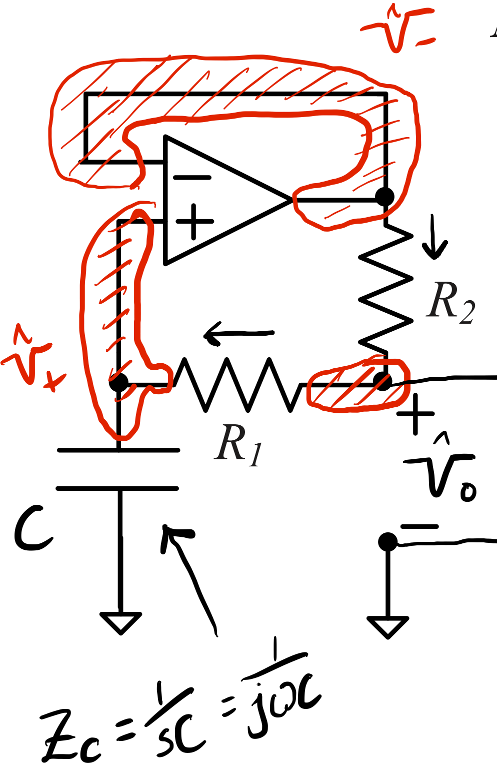


While not common, negative resistors do occur. Fluorescent lamps have negative differential resistance which is electrically balanced with a ballast which adds positive impedance to the circuit. The ballast counteracts the negative resistance of the tube, thus limiting the current in the lamp.

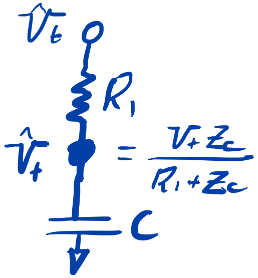
2.2 The Capacitance Multiplier



The circuit above can be used to increase the apparent value of the capacitor, C, as measured at the output terminals labeled $+$ and $-$. Again, we will analyse the circuit using impedance analysis and calculate the output impedance with the application of a test voltage source.



Applying KCL at the output node: $\sum i_{IN} + \sum i_{OUT} = 0$



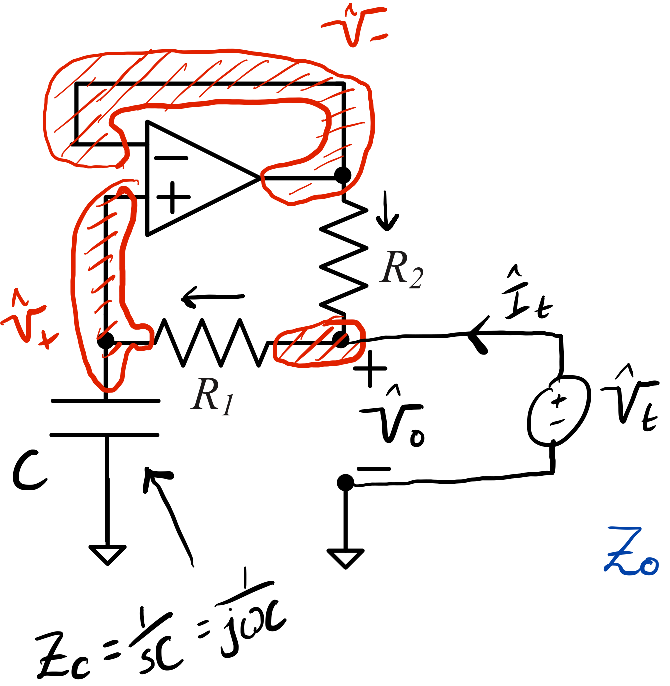
$$\hat{I}_t + \left(\frac{\hat{V}_- - \hat{V}_t}{R_2}\right) - \left(\frac{\hat{V}_t - \hat{V}_+}{R_1}\right) = 0$$

Negative feedback:

$$\hat{V}^- \approx \hat{V}^+ = \hat{V}_t\left(\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R_1}\right) = \frac{\hat{V}_t}{1 + j\omega R_1 C}$$

$$\hat{I}_t + \frac{\hat{V}_t}{R_2(1 + \jmath\omega R_1 C)} - \frac{\hat{V}_t}{R_2} - \frac{\hat{V}_t}{R_1} + \frac{\hat{V}_t}{R_1(1 + \jmath\omega R_1 C)} = 0$$

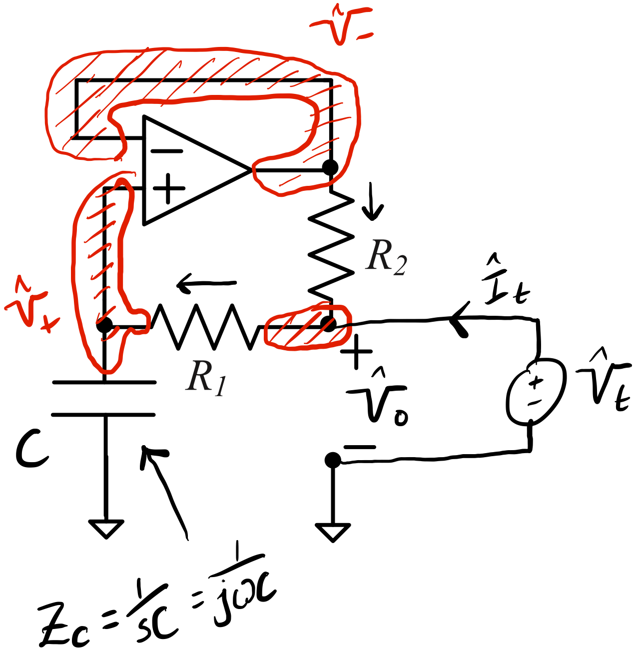
$$\hat{V}_t\left(-\frac{1}{R_2(1 + j\omega R_1 C)} + \frac{1}{R_2} + \frac{1}{R_1} - \frac{1}{R_1(1 + j\omega R_1 C)}\right) = \hat{I}_t$$



$$\frac{1}{\hat{Z}_O} = \frac{\hat{I}_t}{\hat{V}_t} = \frac{-R_1 + R_1(1 + \jmath\omega R_1 C) + R_2(1 + \jmath\omega R_1 C) - R_2}{R_1 R_2(1 + \jmath\omega R_1 C)}$$

$$\frac{1}{Z_o} = \frac{j\omega R_1^2 C + j\omega R_1 R_2 C}{R_1 R_2 (1 + j\omega R_1 C)} = \frac{j\omega C(R_1 + R_2)}{R_2(1 + j\omega R_1 C)}$$

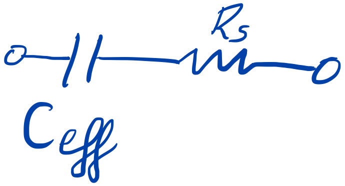
$$Z_o = \frac{R_2(1 + j\omega R_1 C)}{j\omega C(R_1 + R_2)} = \frac{R_2}{j\omega C(R_1 + R_2)} + \frac{R_1 R_2}{R_1 + R_2}$$



$$Z_o = \frac{R_2}{j\omega C(R_1+R_2)} + \frac{R_1 R_2}{R_1 + R_2}$$

This expression can be written in terms of a series-connected capacitor, C_{eff}, and resistor, R_S:

$$\hat{Z}_O = \frac{1}{\jmath\omega C_{eff}} + R_S$$



$$C_{eff} = \frac{C(R_1+R_2)}{R_2} \qquad R_s = \frac{R_1 R_2}{R_1+R_2}$$

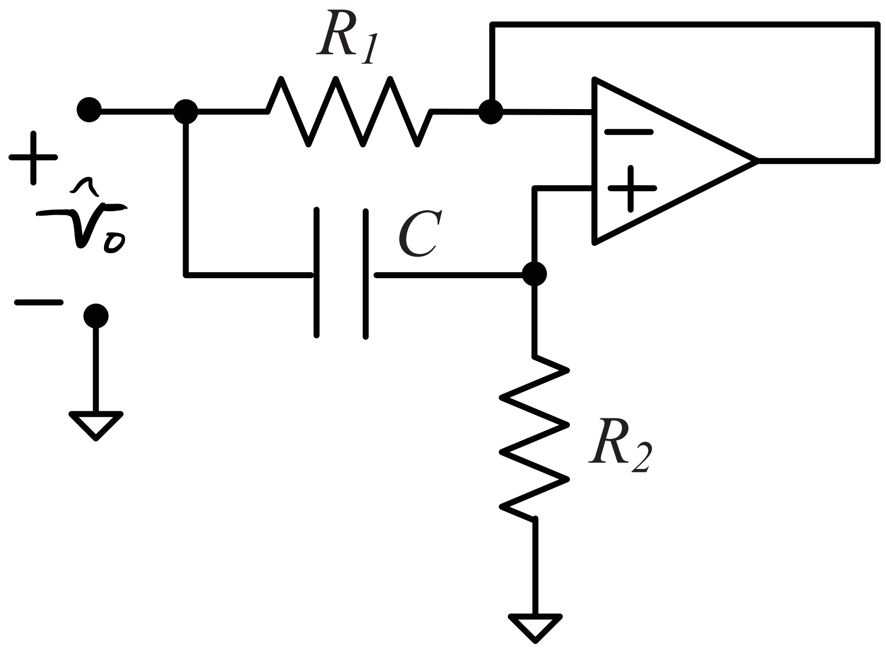
The effective output impedance, $\hat{Z}_O$, can be simplified if $R_1 \gg R_2$:

$$C_{eff} \simeq C\frac{R_1}{R_2} \qquad R_s = R_2$$

Capacitance Multiplier

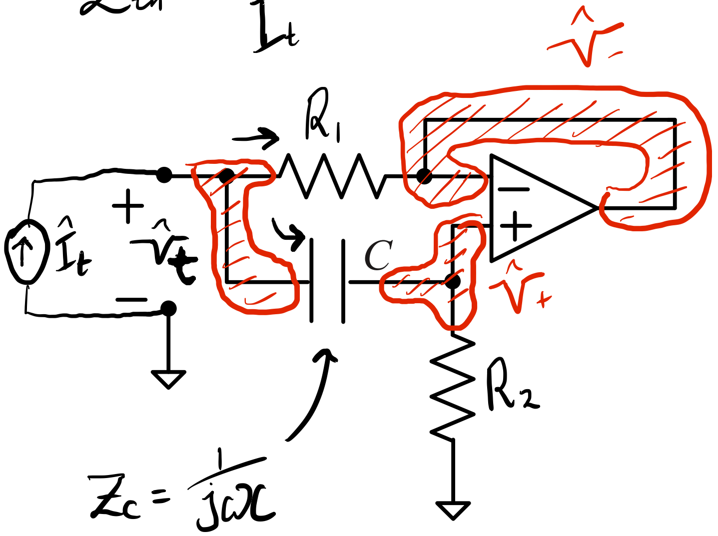
3 The Gyrator Inductor

The inductor is not an easy or cheap device to manufacture and, in some cases, can be substituted by an active op implementation which mimics its properties.



Again, we analyse the circuit using complex analysis and calculate the apparent input impedance, $\hat{Z}_{in}$.

$$\hat{Z}_{in} = \frac{\hat{V}_t}{\hat{I}_t}$$



KCL at the input node: $\sum i_{IN} + \sum i_{OUT} = 0$

$$\hat{I}_t - \left(\frac{\hat{V}_t - \hat{V}_-}{R_1}\right) - \left(\frac{\hat{V}_t - \hat{V}_+}{\left(\frac{1}{j\omega C}\right)}\right) = 0$$

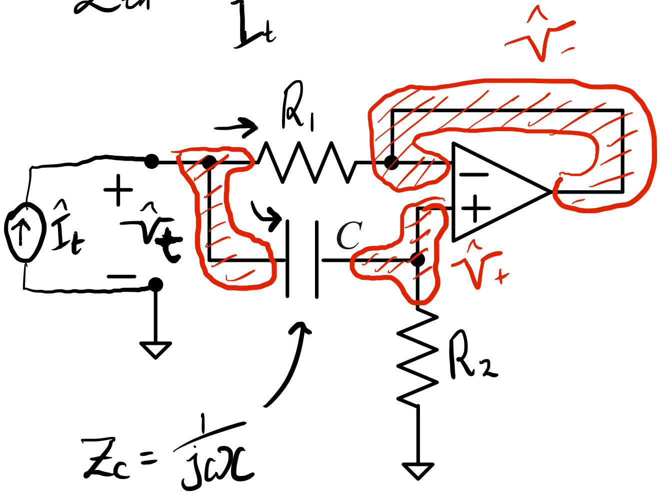
$$\hat{I}_t - \frac{\hat{V}_t}{R_1} + \frac{\hat{V}^-}{R_1} - \jmath\omega C\hat{V}_t + \jmath\omega C\hat{V}^+ = 0$$

But $\hat{V}_- \simeq \hat{V}_+ = \hat{V}_t\left(\frac{R_2}{R_2 + \frac{1}{j\omega C}}\right)$

$$\Rightarrow \quad \hat{I}_t - \frac{\hat{V}_t}{R_1} + \frac{\jmath\omega R_2 C\hat{V}_t}{R_1(1 + \jmath\omega R_2 C)} - \jmath\omega C\hat{V}_t + \frac{\jmath\omega C(\jmath\omega R_2 C)\hat{V}_t}{(1 + \jmath\omega R_2 C)} = 0$$

$$\frac{\hat{I}_t}{\hat{V}_t} = \frac{(1 + \jmath\omega R_2C) - \jmath\omega R_2C + \jmath\omega C R_1(1 + \jmath\omega R_2C) - \jmath\omega R_1C(\jmath\omega R_2C)}{R_1(1 + \jmath\omega R_2C)}$$

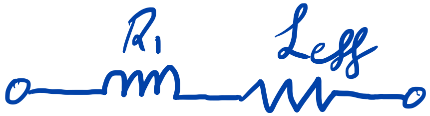
$$\hat{Z}_{in} = \frac{\hat{V}_t}{\hat{I}_t}$$



$$\frac{\hat{I}_t}{\hat{V}_t} = \frac{1}{\hat{Z}_{IN}} = \frac{1 + j\omega R_1 C}{R_1(1 + j\omega R_2 C)}$$

$$\hat{Z}_{IN} = \frac{R_1(1 + j\omega R_2 C)}{1 + j\omega R_1 C} = \frac{R_1}{1 + j\omega R_1 C} + \frac{j\omega R_1 R_2 C}{1 + j\omega R_1 C}$$

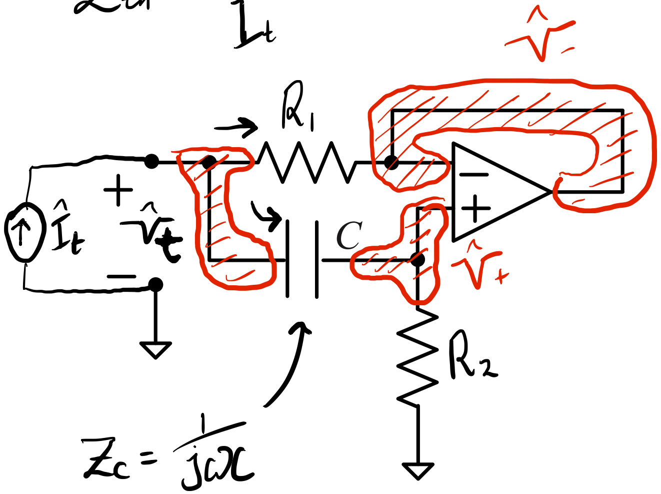
for $\omega \ll \frac{1}{R_1 C} \Rightarrow Z_{in} \simeq R_1 + j\omega R_1 R_2 C$



$$L_{eff} = R_1 R_2 C$$

$$\frac{\hat{I}_t}{\hat{V}_t} = \frac{(1 + \jmath\omega \cancel{R_2C}) - \jmath\omega \cancel{R_2C} + \jmath\omega C R_1(1 + \jmath\omega \cancel{R_2C}) - \jmath\omega R_1 C(\jmath\omega \cancel{R_2C})}{R_1(1 + \jmath\omega R_2 C)}$$

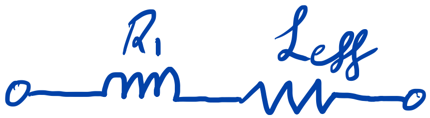
$$\hat{Z}_{in} = \frac{\hat{V}_t}{\hat{I}_t}$$



$$\frac{\hat{I}_t}{\hat{V}_t} = \frac{1}{\hat{Z}_{IN}} = \frac{1 + j\omega R_1 C}{R_1(1 + j\omega R_2 C)}$$

$$\hat{Z}_{IN} = \frac{R_1(1 + j\omega R_2 C)}{1 + j\omega R_1 C} = \frac{R_1}{1 + j\omega R_1 C} + \frac{j\omega R_1 R_2 C}{1 + j\omega R_1 C}$$

for $\omega << \frac{1}{R_c} \Rightarrow Z_{in} \simeq R_1 + j\omega R_1 R_2 C$



$$L_{eff} = R_1 R_2 C$$

ductor can be selected precisely and simulated inductors have higher accuracy than physical inductors, due to the lower cost of precision capacitors than inductors.

3. **Energy storage**. Synthesised inductors do not have the inherent energy storing properties of real inductors and this limits power applications using the synthesised inductor.
4. **External Fields** The synthesised inductor has no associated magnetic field and does not obey Faraday's Law.
5. **Grounding** The fact that one side of the simulated inductor is grounded restricts applications. Real inductors are floating.

The gyrator inductor is used extensively in applications such as graphics equalisers (high Q audio bandpass filters) and data access arrangements (DAAs) in telephone networks.